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Revision Notes for Leaving Cert 2011

Maths

Leaving Cert

Higher Level – Differentiation

Question 6 & 7 Paper 1

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Section 2: Calculus

Paper 1 Differentiation: Question 6 and 7

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Differentiation Basics

Our focus here will be to take a quick overview of the fundamentals of Differentiation. Our aim being to clarify any uncertain points. We will also explain the three Differentiation rules.

- Basics
- Product Rule
- Quotient Rule
- Chain Rule

Basics:

Simple Formula: $y = x^N \rightarrow \frac{dy}{dx} = Nx^{n-1}$

At this stage you should already have this in mind. But it's important you also understand what this means in English. All you do is bring down the original power and multiply it by what was differentiated with one taken away from the power.

Question 1: What if y equals more than one number? If y equals more than one number than you just differentiate them both separately and add them together.

EG: $y = x^5 + x^3 \rightarrow \frac{dy}{dx} = 5x^4 + 3x^2$

Question 2: What if y equals a number without x (a constant)? Well then $\frac{dy}{dx}$ =0. If y = 8, the assume that is in fact y = $8x^0$, then 0 is brought down and multiplied. And anything that is multiplied by 0 is obviously 0.

EG: $y = 9 \rightarrow \frac{dy}{dx} = 0$

Question 3: What if x doesn't have a power? Like above, remember that x is in fact x^1. Then when you take away 1 from the power you will get x^0 which equals 1.

EG: y = 4x ➔ $\frac{dy}{dx}$ = $4(1)x^0 = 4$

Question 4: What about if I am dividing by x instead of multiplying? If x you are dividing by x (x below the line) then you must bring it up before you differentiate. Remember that this will multiply the power by a negative sign when you bring it up.

EG: $y = \frac{7}{x^3} = 7x^{-3} \rightarrow \frac{dy}{dx} = 7(-3)x^{-4} = -21x^{-4}$

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The Quotient Rule: $\frac{V.\frac{du}{dx} - U.\frac{dv}{dx}}{V^2}$

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WARNING! Keep an eye out for that minus sign in the middle of the formula. It is easy to only apply it to the first term it's in front of rather than all of the numbers it is ahead of. The minus sign also means that you cannot mix the order up. It must be: $V.\ \frac{du}{dx} - U.\frac{dv}{dx}$ and not $U.\frac{dv}{dx} - V.\ \frac{du}{dx}$

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WARNING! In the formula, $\frac{dy}{dx} = \frac{dy}{dt} X \frac{dt}{dx}$. Remember to flip $\frac{dx}{dt}$ to $\frac{dt}{dx}$!This is as simple as just putting what is on top to the bottom and what is at the bottom to the top.

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WARNING! In the table book all the formula feature something like, $\sin x \rightarrow \cos x$ etc.

But there is a part which isn't apparent there: $\sin x \rightarrow \cos x . \frac{dy}{dx}$.

However, when y = x, then $\frac{dy}{dx}$ =1. So it doesn't affect the formula. This is why step three multiplies by $\frac{du}{dx}$.

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Table Book, p.16

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$\frac{dy}{dx} > 0$, ➔*increasing*

$\frac{dy}{dx} < 0$ ➔*decreasing*

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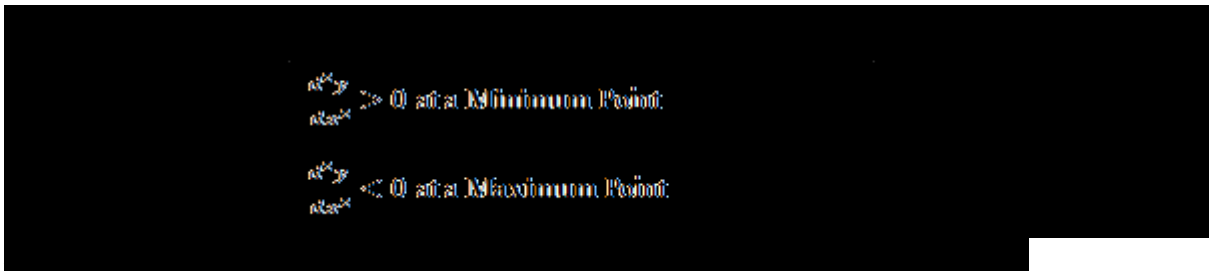

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WARNING! It is easy to forget to find the y value at the end but it is vital that you do so.

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Inflection points occur when $\frac{d^2y}{dx^2}=0$

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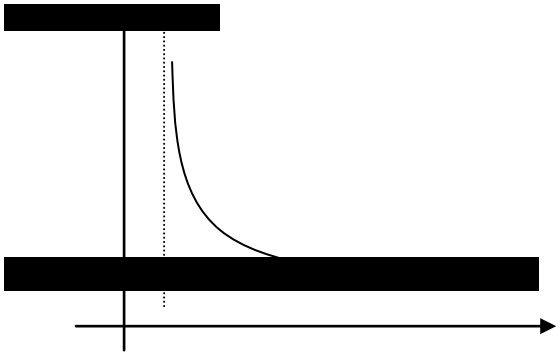

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(iv) Point of inflection

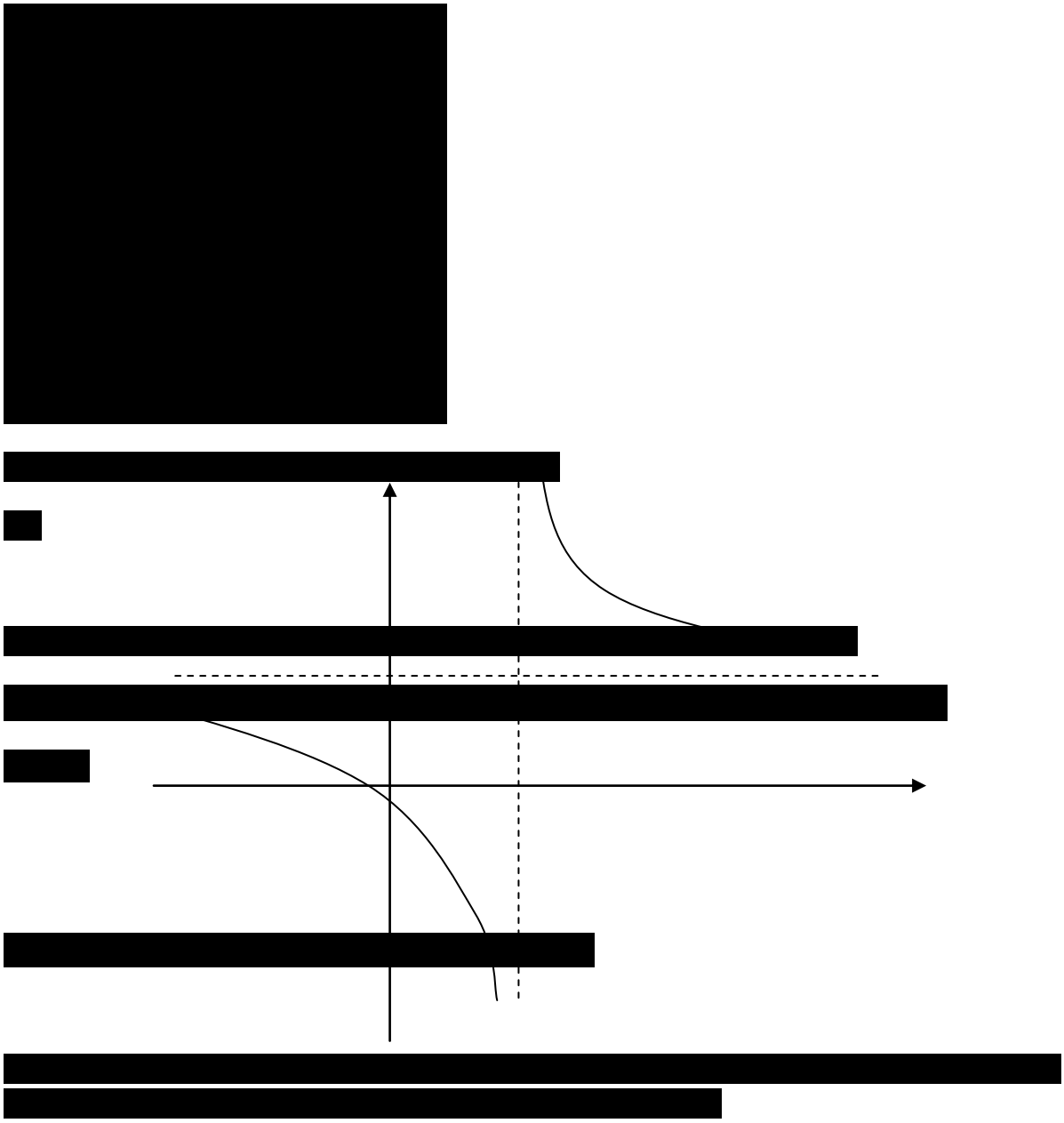
Since $\frac{dy}{dx} = 3x^2 - 12x + 9$

$\therefore \frac{d^2y}{dx^2} = 6x - 12$ {Pt. of inflection when $\frac{d^2y}{dx^2} = 0$}

$\Rightarrow 6x - 12 = 0$

$\Rightarrow x = 2$

If $x = 2$ then $y = 2^3 - 6(2)^2 + 9(2)$



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If $f(x) > 0$ *and* $f(y) < 0$, then there must be a root between x and y.

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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WARNING! You are asked to find the speed here, do not think that S stands for speed, it stands for distance. Pay particular attention to this as it is designed to slip you up!

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WARNING! In no other questions are you required to insert a unit of measurement, but if you do not do so for velocity and acceleration then you will lose marks!

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$$y = \left(x^2 - 3x\right)^3$$

$$\frac{dy}{dx} = 3\left(x^2 - 3x\right)^2 \left(2x - 3\right)$$

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