mocks.ie

Revision Notes for Leaving Cert 2011

Maths

Leaving Cert

Higher Level –Algebra

Paper 1

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Section 1: Algebra

Paper 1, Question 1 and 2

Table of Contents:

Basic Rules for Manipulation:

To begin it's important that you are 100% clear on the fundamentals of manipulation. You will have seen these for years but even then you may not have understood them. They will be short and simple but it's important you are wholly certain of each step. We will cover:

- Fraction below the line
- Adding fractions
- Manipulating Equations

Example: Fraction below the line

Simplify: $\frac{5}{\frac{4}{X}}$

$\frac{4}{x} \rightarrow \frac{x}{4}$ {Take the fraction underneath the line and turn it upside down}

$5 \times \left(\frac{x}{4}\right) = \frac{5x}{4}$ ← Multiply the new fraction by what was on top of the line. Now you have gotten rid of the fraction below the line

Example: Adding Fractions

$\frac{1}{3} + \frac{11}{4} = x$

(3)(4) {Take what is underneath the lines and multiply them by each other}

$\frac{(3)(4)1}{3} + \frac{(3)(4)11}{4} = (3)(4)x$ {Multiply each fraction by (3)(4) }

$\frac{\cancel{(3)}(4)1}{\cancel{3}} + \frac{(3)\cancel{(4)}11}{4} = (3)(4)x$

$4 + 33 = 12x \rightarrow 37 = 12x$

WARNING! Remember that if you multiply on the left you must do the same to the right. So (3)(4) are multiplied by the X as well. Don't forget. And if there was a fraction on the other side of the equation I would take what is below the line there as well.

Important Formula: This Formula works for adding fractions::

$\frac{a}{b} + \frac{c}{d} = \frac{a(d)+c(b)}{bd}$

WARNING! Not in the table book, but you don't need to learn it really, just know how to apply it.

Example: Manipulating Equations

Simplify $\sqrt[3]{\frac{3x-1}{2x+1}} = y$, express x in terms of y.

$$\sqrt[3]{\frac{3x-1}{2x+1}} = y$$

$$\Rightarrow \left(\frac{3x-1}{2x+1}\right)^{\frac{1}{3}} = y$$

$$\Rightarrow \left[\left(\frac{3x-1}{2x+1}\right)^{\frac{1}{3}}\right]^3 = [y]^3 \qquad \text{cube both sides}$$

$$\Rightarrow \frac{3x-1}{2x+1} = y^3$$

$$\Rightarrow 3x-1 = y^3(2x+1) \qquad \text{multiply across by base}$$

$$\Rightarrow 3x-1 = 2xy^3 + y^3$$

$$\Rightarrow 3x - 2xy^3 = 1 + y^3 \qquad \text{rearrange so that all } x\text{'s on left}$$

$$\Rightarrow x(3-2y^3) = 1+y^3 \qquad \text{factorise}$$

$$\Rightarrow x = \frac{1+y^3}{3-2y^3}$$

This is a skill which it is vital that you learn. Adjusting the formula/equation to suit your purpose is something you will be required to do again and again. The key focus is to make sure that whatever you do to one side you do to the other.

Simultaneous Equations

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WARNING! Ensure all signs are changed when taking away. Simple Slip!

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CHECK STEP: Sub your x and y value into the second equation to ensure you are correct

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[redacted]

[redacted]

[redacted]

[redacted]

Remove **<u>the same variable</u>** from one of the same equations and the other you didn't use.

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[redacted]

[redacted]

[redacted]

CHECK STEP: As before, simply sub all these values into one of the other equations you were given. If that works then all are correct.

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Simplifying: Surds and Modulus

[redacted]

The Conjugate is a fancy term for a simple action. Just change the sign between the two numbers from + to − (vice versa)

[redacted]

WARNING! This question can get more complicated if there is a surd on top and below the line (E.g. $\frac{6+\sqrt{3}}{4+\sqrt{3}}$.) If this happens it will change nothing. Just get your Conjugate from below the line as usual!

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WARNING!! Check answers. Both may not be correct. Sub them into the original equation to be sure.

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Modulus:

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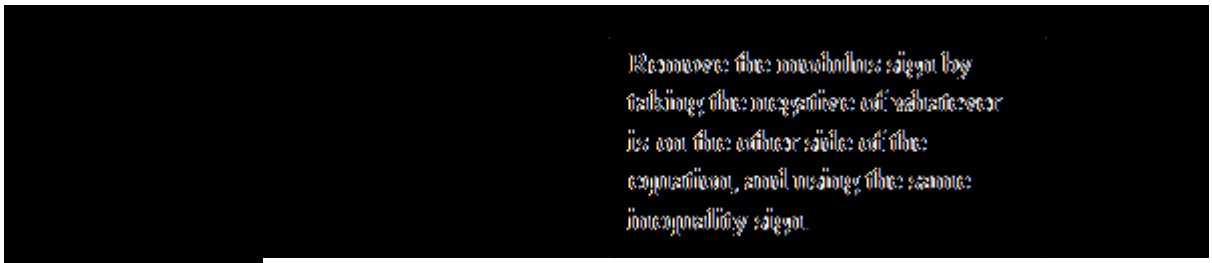

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Alpha and Beta:

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For a Quadratic Equation where $ax^2 + bx + c = 0$

- $\alpha + b = \frac{-b}{a}$
- $\alpha\beta = \frac{c}{a}$

WARNING! These aren't in the table book. Learn them!

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$x^2 - (add\ the\ roots)x + (multiply\ the\ roots) = 0$

Not in the table book!

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WARNING! Make sure you use the roots you were asked to $(\frac{1}{\alpha}, \frac{1}{\beta})$ not the roots of the equation you already have: (α, β)

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Roots of a Quadratic Equation

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1) **2 Real Distinct Roots:** $b^2 - 4ac > 0$
2) **2 Equal Real Roots:** $b^2 - 4ac = 0$
3) **2 Roots aren't real:** $b^2 - 4ac < 0$

These comes from the $-b\ formula: x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

WARNING! Not in the table book

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Nature of Roots on a Graph

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Factors and Roots

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Factor theorem: $(x-k)$ **is a factor of the polynomial** $f(x)$ **if and only if** $f(k)=0$

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WARNING: Make sure you state that as you divided by a factor the two equations equal each other as the remainder equals 0.

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Algebra Proofs:

Important Formulae:

[redacted]

- $(x + y)^2 = x^2 + 2xy + y^2$
  $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$
  $(x - y)^3 = x^3 - y^3 + 3xy^2 - 3yx^2$
- $x^2 - y^2 = (x - y)(x + y)$
- $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$
  $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$

WARNING! Not in the table book:

[redacted]