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Revision Notes for Leaving Cert 2011

Maths

Leaving Cert

Higher Level – Integration

Question 8 Paper 1

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Integration Paper 1, Q8

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Integration Basics:

We will begin our review of Integration with a quick overview of the basics. We will focus on clarifying the simple points which you may have overlooked. It's important that you are certain on the simple steps as that can make the more difficult questions easier.

Basics:

The basic rule of integration is: *(Page 26, Table Book)*

$$\int x^N dx = \frac{x^{N+1}}{N+1}$$

Meaning you take the power of X add 1 to it and then divide by that new power.

Question 1: How is Integration the reverse of differentiation? The basic idea behind integration is the following: $\int \frac{dy}{dx} = y + C$ When you differentiate y you get $\frac{dy}{dx}$. When you integrate $\frac{dy}{dx}$ you get y.

You will find that all the rules below are very similar to Differentiation.

Question 2: What if I am asked to integrate x^n with a number in front of it? You integrate as you usually would without the number and then multiply by that number.

In fact, we can do the following: $\int ax^n dx = a\int x^n dx$ where a is a constant (normal number).

Question 3: What if I am asked to integrate a number without an x? Just like in Differentiation, you think of a number without an x as a number multiplied by x^0 because $x^0 = 1$. Then just follow the formula above.

Question 4: What if I am asked to integrate two numbers added to each other? Again, just as in Differentiation, you integrate the terms separately and then add them together again.

Question 5: How do I deal with a Square root? You just look at the square root as $x^{\frac{1}{2}}$ and integrate normally then.

Question 6: How to I deal with an X below the line?

Think of what you would do for Differentiation. Just use indices to bring the X above the line and integrate normally.

Definite and Indefinite Integration:

We will now look at the two fundamental types of integrations: Definite and Indefinite. They are not all that different but quite easy to identify.

$\int x^N dx$ This is an Indefinite Integral

As you can see above the only difference is that the integration sign on the Definite Integral has a number above and below it.

Indefinite Integration: Constants of Integration

The Constant of Integration may sound scary but it's just a "+ C". After every indefinite integration you must add C to your answer.

You'll recall that we claimed that Integration was the reverse of Differentiation. Now is a good time to really test that. Here are two questions:

$$y = x^3 \Longrightarrow \frac{dy}{dx} = 3x^2 \qquad y = x^3 + 15 \Longrightarrow \frac{dy}{dx} = 3x^2$$

$$\int 3x^2 \Longrightarrow y = x^3 \qquad \int 3x^2 \Longrightarrow y = x^3$$

You'll notice that although my beginning y values are different my final answer for y is the same. This is clearly wrong. As differentiation gets rid of constants (like the 15 above) Integration cannot take account of them. So instead we add a "+C" which means any number both positive and negative could have been part of the original integration.

So the example from above should read:

$$y = x^3 + 15 \Longrightarrow \frac{dy}{dx} = 3x^2$$

$$\int 3x^2 \Longrightarrow y = x^3 + C$$

WARNING! Not including a "+C" is a sure way to lose marks. So make sure you include it for every Indefinite Integration

Definite Integration:

As you've seen above you can tell a Definite Integration question from an Indefinite Integration Question because a Definite Integration sign will have a number above and below it. But there are other differences two:

- Definite Integration does not have a "+C" ever!
- With Definite Integration you will get an answer of numbers

A Definite Integration works something like this: $\int_b^a f(x)dx = \int f(a) - \int f(b)$

In short, you integrate, sub in the top value into that integration and then take the bottom value subbed into the integration away from it. These values are called 'limits'.

Example: Definite Integration

$\int_0^1 \left(1-\sqrt{x}\right)^2 dx$ Simplify first

$\int_0^1 \left(1-2\sqrt{x}+x\right) dx$

$\int_0^1 1-2x^{\frac{1}{2}}+x\, dx$ Now integrate

$\left[x-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^2}{2}\right]_0^1$

$\left[x-\frac{4x^{\frac{3}{2}}}{3}+\frac{x^2}{2}\right]_0^1$ Substitute $x=1$ minus $x=0$

$\left[1-\frac{4(1)^{\frac{2}{3}}}{3}+\frac{(1)^2}{2}\right]-\left[0-\frac{4(0)^{\frac{2}{3}}}{3}+\frac{0^2}{2}\right]$

$\left[1-\frac{4}{3}+\frac{1}{2}\right]-[0]$

$\frac{1}{6}$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

With powers, select whichever part has the highest power. Then let U equal the numbers inside the bracket of highest power

[redacted]

[redacted]

[redacted]

[redacted]

WARNING! Don't forget to sub the value back in

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\int Cos(ax)dx = \frac{Sin(ax)}{a}$$

$$\int Sin(ax+l)dx = -\frac{Cos(ax+l)}{a}$$

[redacted]

[redacted]

[redacted]

[redacted]

$$2\cos A\cos B = \cos(A+B)+\cos(A-B)$$
$$2\sin A\cos B = \sin(A+B)+\sin(A-B)$$
$$2\sin A\sin B = \cos(A-B)-\cos(A+B)$$
$$2\cos A\sin B = \sin(A+B)-\sin(A-B)$$

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[redacted]

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[redacted]

For these questions always let U equal the function which is to the highest power

[redacted]

[redacted]

$$\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

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[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Divide the number multiplied by x by 2 ($\frac{6}{2} = 3$). This is how you know what to square.

[redacted]

[redacted]

[redacted]

Again, just divide 4 by 2 and you'll get 2. Therefore it must be $(x+2)^2$

[redacted]

$\sqrt{a^2-x^2}$

Let $x = a\sin\theta \Longrightarrow x^2 = a^2\sin^2\theta$

[redacted]

Integrate $\int_0^3 \sqrt{9-x^2}\,dx$

STEP ONE: Simplify

$\sqrt{9-x^2}\,dx = \sqrt{(3)^2-x^2}\,dx$ Let x $=3\sin\theta$ ← $x = a\sin\theta$

$\sqrt{(3)^2-9\sin^2\theta}\,dx$ dx=$-3\cos\theta d\theta$

$\sqrt{9-9\sin^2\theta}\ (-3\cos\theta d\theta)$

$\sqrt{9(1-\sin^2\theta)}\ (-3\cos\theta d\theta)$ $(1-\sin^2\theta)=\cos^2\theta$ Table book, p.13

$\sqrt{9\cos^2\theta}(-3\cos\theta d\theta)$

$3\cos\theta(-3\cos\theta d\theta) = -9\cos^2\theta d\theta$

[redacted]

[redacted]

[redacted]

[redacted]

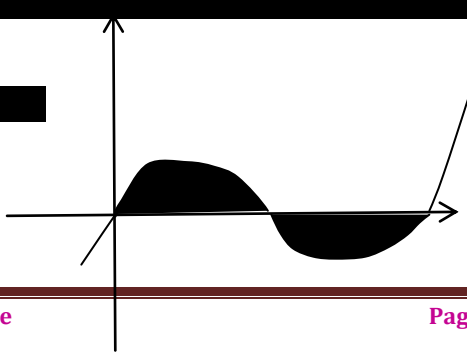
$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

Table Book, p. 14

$$Area = \int_a^b f(x)dx$$

Where f(x) is the curve.

b, a are the points you want the area between.



Area below the graph gives negative value. Note the absolute value signs.

$$Area = \int_a^b f(y)dx$$

Where f(y) is the curve.

b, a are the points you want the area between.

As it is on the y-axis, y=b, y=a

[redacted]

Area= $\int_a^b f(x)dx - \int_a^b g(x)dx$

Where f(x), g(x) are the two curves/curve and a line

b, a are the points where they intersect

[redacted]

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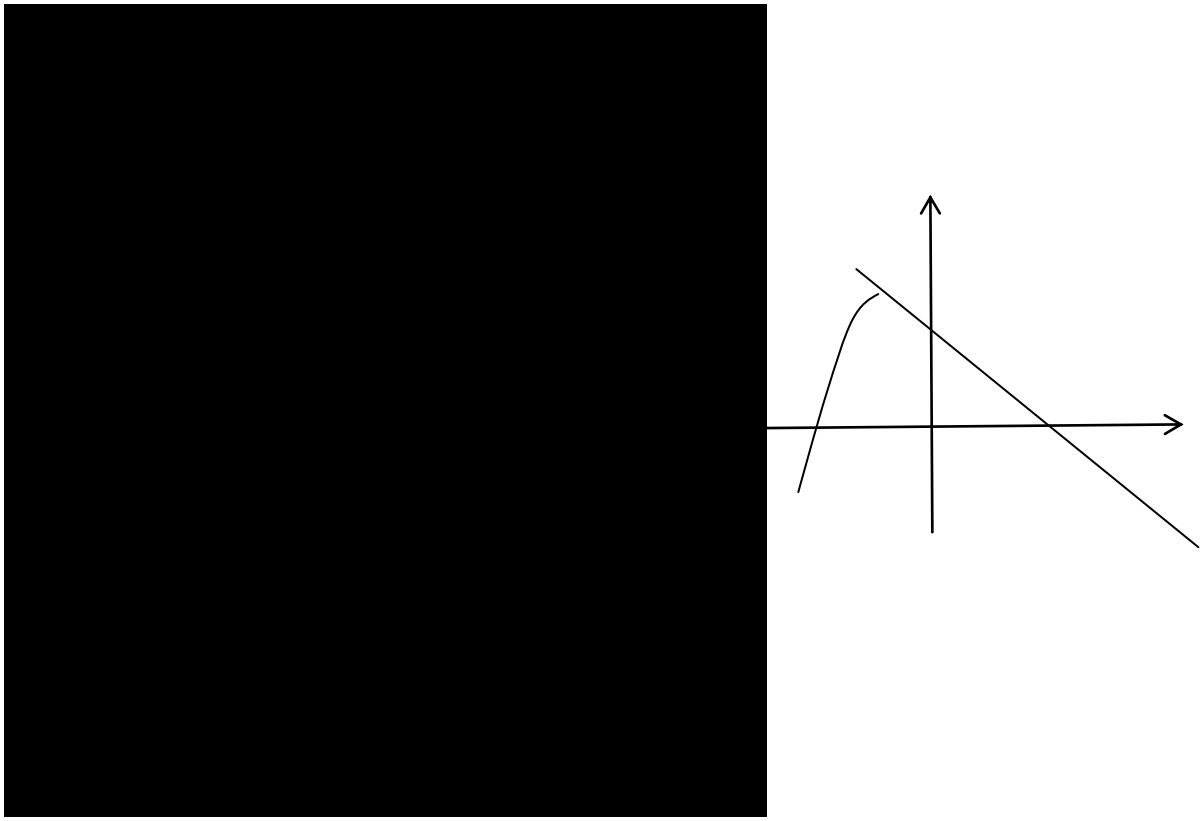

WARNING! Absolute value signs should not go in here. They are only put in when you are adding these numbers together in the last line!