Revision Notes for Leaving Cert 2011

mocks.ie

Maths

Leaving Cert

Higher Level – Differentiation

Question 6 & 7 Paper 1

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Paper II Co-ordinate Geometry: Question 1

Paper II Q.1 and 3

Co-ordinate Geometry

This is a very popular combination and most students attempt one if not both of these question. If you are comfortable with co-ordinate geometry of the line then you should do the question on the circle. The questions asked in the circle rely heavily on the formula and techniques learnt in the line.

The Line Q.3

This question can be broken down into three sections.

1. Basic questions involving the use of the equation of the line formulae.
2. Parametric Equations
3. Linear Transformations

1. Basic questions involving the formulae given.

One of the biggest challenges in the past was the fact that the formulae had to be learnt. Therefore, mistakes here would lead to incorrect answers and to questions that would not work out. Now however the formulae are all given in the table book page 18 & 19. Familiarise yourself with layout and format of these formulae it can be different to your text book.

With that said the formula for concurrent lines is not given

> If $L_1 : a_1x + b_1y + c_1 = 0$ and $L_2 : a_2x + b_2y + c_2 = 0$ are the equation of two non-parallel lines then the equation
>
> $$\mu(a_1x + b_1y + c_1) + \lambda(a_2x + b_2y + c_2) = 0$$
>
> Represents all lines concurrent with L_1 and L_2.

Also note that you must learn the proof for 'the perpendicular distance formula' and for 'the angle between two lines formula'.

The secret to this topic is to think through the question, formulate a plan and make sure that you do not jump to conclusions, such as assuming that lines are parallel or perpendicular. Remember all diagrams are not to scale and can't be measured as part of your answer.

You should try to approach all questions on the line in the same manner,

1. Read the question carefully and identify what is being asked,
2. Draw a sketch of the information given, this can be a big help!
3. Decide what formulae to use.
4. Formulate a plan before you start.
5. Take your time and make sure that you don't make silly mistakes

E.g. The equation of the line L is $14x+6y+1=0$. {2000, Paper II Q.3 (a)}
Find the equation of the line perpendicular to L that contains the point $(-3,2)$.

Ans. $y-y_1=m(x-x_1)$ ……….[formula to find the equation of a line]

Note: Think about what you need for this formula.
Need a point ….given $(-3,2)$
and the slope … not given!
Therefore, need to find this first.

L: $14x+6y+1=0$

$6y=-14x-1$

$y=\frac{-14}{6}x-\frac{1}{6}$

[redacted]

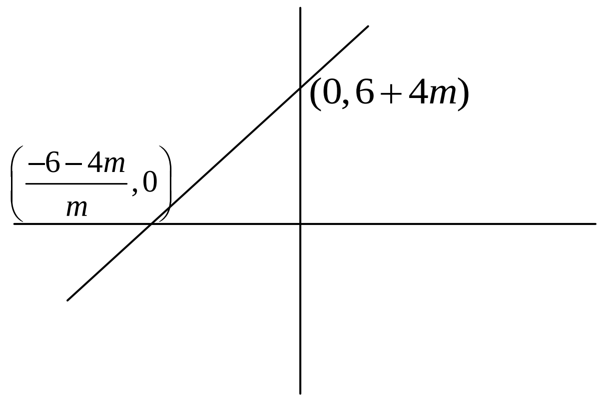
Same as above. Identify the formula needed and apply it.

[redacted]

Note: This is all the same as in the Junior Cert. Don't be put off by the m's. Just remember to do what you have always done.

[redacted]

Note: a quick sketch of the question always helps. Now we can see that we have the 3 vertices and one is $(0,0)$.



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Note: as with the last two examples find the equation using Junior Cert methods, don't worry about the second line for the moment it will turn up.

Find the points as above.

Remember all information in the question must be used so substitute values into the given equation.

Now we have two values for *m* as asked!

Parametric equations of a line

[redacted]

Note: Re-write each equation in terms of *t*. Therefore we can let them equal and write an equation in terms of *x* and *y*.

Linear Transformations

[redacted]

$$x' = ax + by$$
$$y' = cx + dy$$

Note: There is a separate rule for moving the *x* and the *y* coordinates

[redacted]

[redacted]

$$x' = x + 2y$$

$$y' = -x - y$$

[redacted] $f(a),\ f(b),\ f(c),\ f(d)$

[redacted]

[redacted] $(x,y) \to (x+2y, -x+y)$

Note: There is a separate rule for moving the *x* and *y* values.

$a(2,-2) \to (2+2(-2), -2-2) = (-2,-4) \quad = f(a)$

$b(4,1) \to (4+2(1), -4+1) = (6,3) \quad = f(b)$

$c(-2,-1) \to (-2+2(-1), -(-2)-1) = (-4,1) \quad = f(c)$

$d(2,5) \to (2+2(5), -2+5) = (12,3) \quad = f(d)$

[redacted]

$\sqrt{(4-2)^2+(1+2)^2} \neq \sqrt{(6+2)^2+(-3+4)^2}$

$\sqrt{13} \neq \sqrt{65}$

$\therefore |ab| \neq |f(a)\, f(b)|$ as asked.

Note: As said earlier, simply do what you are told. Find the length of *ab* and *f* (*a*) to *f* (*b*) and show that they are not the same

[redacted]

$a(2,-2) \to (0,0)$

$b(4,1) \to (2,3)$

$c(-2,-1) \to (-4,1)$

$\therefore$ area $= \frac{1}{2}|2(1)-(-4)(3)|$

$= 7$

$f(a) = (-2,-4) \to (0,0)$

$f(b) = (6,-3) \to (8,1)$

$f(c) = (-4,1) \to (-2,5)$

$\therefore$ area $= \frac{1}{2}|8(5)-(-2)(1)|$

$= 21$

Note: as above but using the area of a triangle formula

[redacted]

[redacted]

Note: Again don't get confused. Use the appropriate formula and simply work through the question

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$g = \left(\frac{2+4-2}{3}, \frac{-2+1-1}{3} \right)$$

[redacted] $= \left(\frac{4}{3}, -\frac{2}{3} \right)$ [redacted]

Note: the centroid does not come up very often and can be overlooked.

A triangle with vertices $a(x_1, y_1),\ b(x_2, y_2),\ c(x_3, y_3)$

has centroid $g = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

[redacted]

[redacted]

$(x', y') = (7, 8)$ [redacted]

$\Rightarrow (x + 2y, -x + y) = (7, 8)$

$\therefore x + 2y = 7 \quad \text{and} \quad -x + y = 8$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Note: Using a simultaneous equation re-write the two equations in terms of x and y.

[redacted]

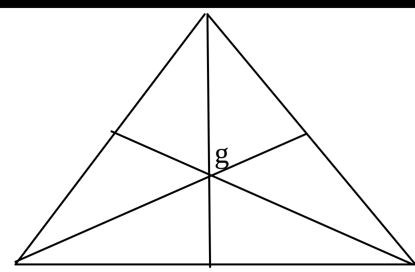
[redacted]

Note: Now we know the image of each x and y value simply replace the x and y values in the equation to find the image of the line.

[redacted]

[redacted]

Note: *g* divides each median in the ratio 2:1. This can be particularly important for the Vectors question.



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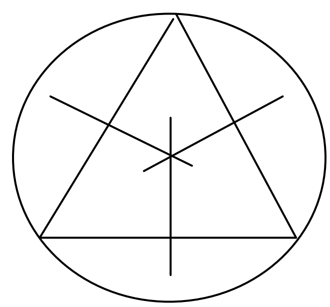

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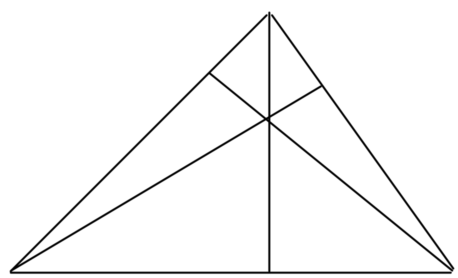

The Circle Q.1

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Note: to find the centre, find $-\frac{1}{2}$ the *x*-coefficient and $-\frac{1}{2}$ the *y*-coefficient

[redacted]

[redacted]

Note: Centre is $(0,0)$ so use the first formula.

Note: If you draw a quick sketch you will find that you are back to co-ordinate geometry of the line using the perpendicular distance formula.

[redacted]

Note: Centre is not $(0,0)$ so must use the formula $(x-h)^2+(y-k)^2=r^2$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Note: In practice it is easier to use the quick rule, centre

$$= \left(-\frac{1}{2} \text{ coefficient of } x, \ -\frac{1}{2} \text{ coefficient of } y \right)$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Note: Substitute each of the points into the general equation of a circle.

This will leave you with three equations in terms of g, f and c. Now solve using simultaneous equations as in Algebra.

[redacted]

[redacted]

Note: This question is taken from 2002 Paper II Q.1 (b) (ii). It can be solved in a few different ways but the method, although slightly longer, will always work.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

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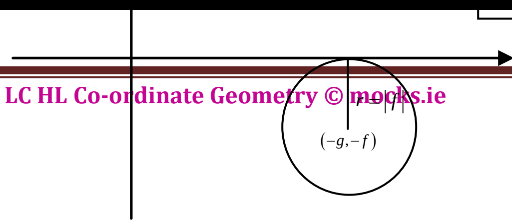


[redacted]

[redacted]

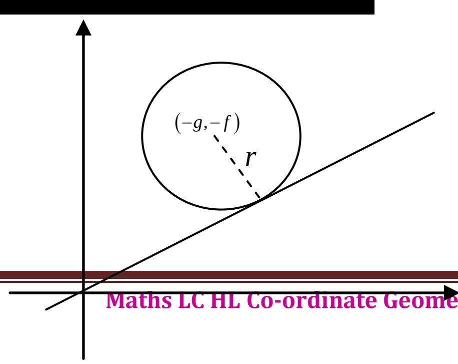
Note: As before the question is reduced down to three pieces of information and from this three equations can be found.

[redacted]

[redacted]

[redacted]

[redacted]



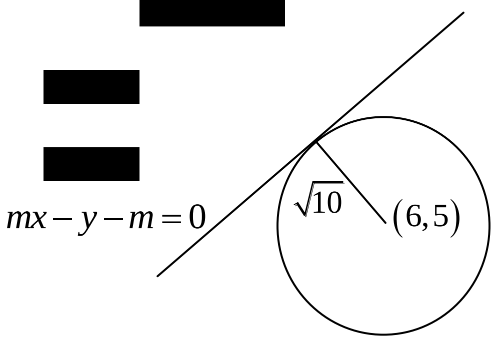
radius $= \perp$ distance $= \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

2. Finding the tangent to a circle.

[redacted]

[redacted]

[redacted]



Firstly draw a sketch of the question. Notice that the perpendicular distance for the centre (which we can find) is equal to the radius (which we can also find).

As always just focus on finding one tangent. The other one will turn up.

Now find the equation of the line in terms of *m*.

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[redacted]

[redacted]

3. Touching circles and intersection between circles and lines

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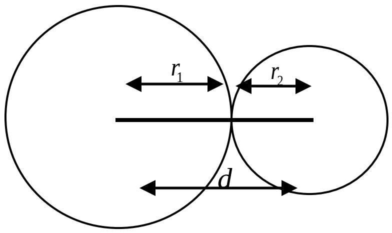
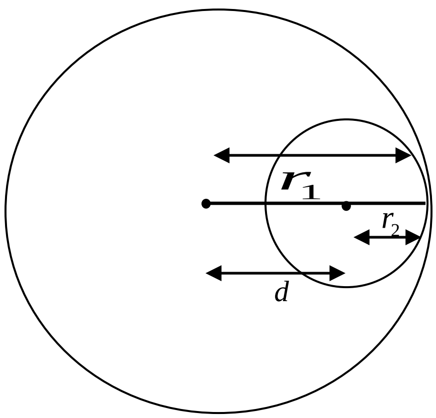


Therefore, $d = r_1 + r_2$

[redacted]



Therefore, $d = r_1 - r_2$

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[redacted]

4. Parametric equation of a circle.

[redacted]

[redacted]

Note: By writing t in terms of x we can substitute this into our second equation. This gives us an equation in terms of x and y only. Allowing us to reform the Cartesian equation and hence find the radius.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Note: Firstly write each equation in terms of $\cos\theta$ and $\sin\theta$.

Then substitute these values into $\cos^2\theta + \sin^2\theta = 1$ to get the equation of the circle.