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Revision Notes for Leaving Cert 2011

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Maths

Leaving Cert

Higher Level – Vectors

Question 2 Paper 2

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Paper II Q.2 Vectors

The Vectors question is very popular and rightly so. It is a relatively short question allowing you to spend more time on other areas and is one of the easiest questions on Paper II.

It is broken down into two main areas:

1. Geometry based questions, using the properties of vectors
2. More algebra based questions using $\vec{i}$ and $\vec{j}$ vectors in the Cartesian plane.

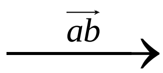
Students find the second section a lot easier as it is very similar to algebra. However, the first section can cause problems and it is very important to take your time and make sure that everything that you do is double checked.

1. Geometry based questions, using the properties of vectors

1.1 Properties of Vectors in the Coordinate plane.

(i) A vector is a translation in a certain direction, a certain distance.

The vector from the point a to the point b is written as $\overrightarrow{ab}$.



(ii) Equality of Vectors

Two vectors $\overrightarrow{ab}$ and $\overrightarrow{cd}$ are said to be equal if they are parallel, have the same length and have the same direction. Therefore, a vector can be moved and still be equal as long as the above facts are still true.

(iii) Negative Vectors

A negative vector is simply a reversal in a vectors direction. $\vec{ab} = -\vec{ba}$, ie the vector $\vec{ab}$ goes from a to b, where as its negative goes from b to a.

Therefore, $\vec{ab} + \vec{ba} = 0$.

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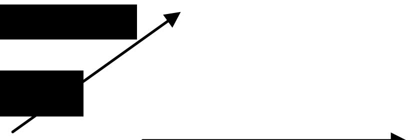

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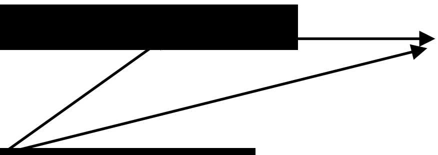

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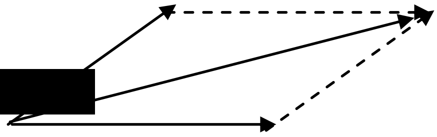

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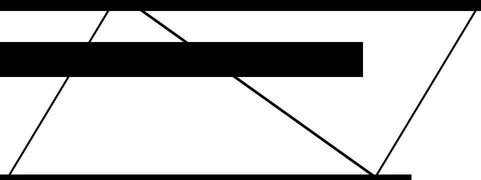

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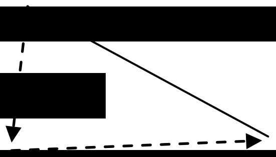

[redacted]

$$\vec{ab} = \vec{ao} + \vec{ob}$$
$$= -\vec{oa} + \vec{ob}$$
$$= \vec{ob} - \vec{oa}$$
$$= \vec{b} - \vec{a}$$

[redacted]

1.2 Parallel Vectors in the Plane

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[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

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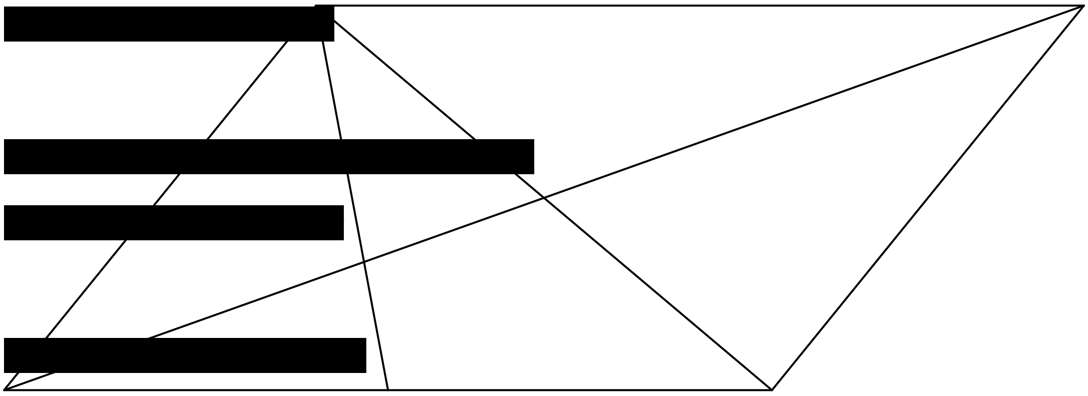


[redacted]

[redacted]

$$\vec{y} = \overrightarrow{oy}$$
$$= \overrightarrow{or} + \overrightarrow{ry}$$
$$= \vec{r} + \tfrac{1}{2}\vec{p}$$

[redacted]

Note: Find the start and end of the vector required. Then work out another path that will take you from the start to the end using the vectors required in the question, i.e. $\vec{p}$ and $\vec{r}$ in this case.

[redacted]

$$\overrightarrow{px} = \tfrac{2}{3}\overrightarrow{py}$$
$$= \tfrac{2}{3}(\vec{y} - \vec{p})$$
$$= \tfrac{2}{3}\vec{y} - \tfrac{2}{3}\vec{p}$$
$$= \tfrac{2}{3}(\vec{r} + \tfrac{1}{2}\vec{p}) - \tfrac{2}{3}\vec{p}$$
$$= \tfrac{2}{3}\vec{r} + \tfrac{1}{3}\vec{p} - \tfrac{2}{3}\vec{p}$$
$$= \tfrac{2}{3}\vec{r} - \tfrac{1}{3}\vec{p}$$

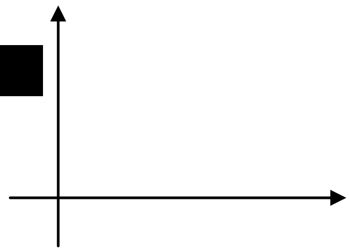
[redacted]

Note: Be careful when given ratios. Remember a line divided in the ratio 1:2 is the same as $\frac{1}{3}$ and $\frac{2}{3}$ of the length of the line.

2. Algebra based questions using and vectors in the Cartesian plane

2.1 Vectors in the Cartesian Plane

[redacted]



Note: Think of it in the same way as an *x* and *y* axis.

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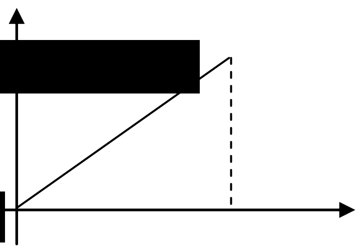

$\vec{v} = x\vec{i} + y\vec{j}$

[redacted]

[redacted]

[redacted]

$$\vec{x}+2\vec{y} = 3\vec{i}+2\vec{j}+2\left(\vec{i}-3\vec{j}\right)$$ [redacted]
$$= 3\vec{i}+2\vec{j}+2\vec{i}-6\vec{j}$$
$$= 5\vec{i}-4\vec{j}$$

$$2\vec{x}-\vec{y} = 2\left(3\vec{i}+2\vec{j}\right)-\left(\vec{i}-3\vec{j}\right)$$
[redacted]
$$= 6\vec{i}+4\vec{j}-\vec{i}+3\vec{j}$$
$$= 5\vec{i}+7\vec{j}$$

[redacted]
$$\overrightarrow{xy} = \vec{y}-\vec{x}$$
$$= \vec{i}-3\vec{j}-\left(3\vec{i}+2\vec{j}\right)$$
$$= -2\vec{i}-5\vec{j}$$

2.2 Equality of Vectors

[redacted]

2.3 Modulus and Unit Vectors

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

2.4 The related perpendicular vector

[redacted]

[redacted]

Note: Parallel vectors have the same direction but can have different lengths, hence we multiply $2\vec{i}+3\vec{j}$ by k.

Similar method for perpendicular vectors

[redacted]

[redacted]

$$\therefore k\left(2\vec{i}+3\vec{j}\right)+l\left(-3\vec{i}+2\vec{j}\right)=-8\vec{i}+\vec{j}$$

$$\Rightarrow 2k\vec{i}+3k\vec{j}-3l\vec{i}+2l\vec{j}=-8\vec{i}+\vec{j}$$

$$\Rightarrow (2k-3l)\vec{i}+(3k+2l)\vec{j}=-8\vec{i}+\vec{j}$$

$\therefore$ using equality of vectors

$\Rightarrow 2k-3l=-8$ and $3k+2l=1$

Solving these equations simultaneously we get $k=-1$ and $l=2$

$\therefore$ The two vectors are

$$k\left(2\vec{i}+3\vec{j}\right)\Rightarrow -1\left(2\vec{i}+3\vec{j}\right)=-2\vec{i}-3\vec{j}$$

$$l\left(-3\vec{i}+2\vec{j}\right)\Rightarrow 2\left(-3\vec{i}+2\vec{j}\right)=-6\vec{i}+4\vec{j}$$

We can check this:

$$-2\vec{i}-3\vec{j}+\left(-6\vec{i}+4\vec{j}\right)=-8\vec{i}+\vec{j}$$

2.5 Dot Product (Scalar product)

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[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

2.6 Properties of the Dot Product

[redacted]

[redacted] $\vec{x}.\vec{y} = \vec{y}.\vec{x}$

[redacted]

[redacted]

$$\left(a\vec{x}+b\vec{y}\right)\left(c\vec{x}+d\vec{y}\right) = a\vec{x}\left(c\vec{x}+d\vec{y}\right)+b\vec{y}\left(c\vec{x}+d\vec{y}\right)$$
$$= ac\vec{x}\vec{x}+\left(ad+bc\right)\vec{x}\vec{y}+bd\vec{y}\vec{y}$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted] $\vec{x}.\vec{x} = \left|\vec{x}\right|.\left|\vec{x}\right|\cos 0$ [redacted]

$$= \left|\vec{x}\right|.\left|\vec{x}\right|$$
$$= \left|\vec{x}\right|^2$$

[redacted]

[redacted]

Note: This is probably the most important property of the dot product.

[redacted]

Note: Don't get caught up in how complicated the question looks. Just do what you are told. There was some confusion here as $\vec{r}$ still contained t values.

[redacted]

Note: See 2.5 above for the dot product rule used here.

[redacted]

Note: If r is on the bisector of $\angle poq$ then $\angle por$ must equal $\angle qor$. You must have a plan before you start this question. As always a quick diagram will always help.

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