Revision Notes for Leaving Cert 2011

Maths

Leaving Cert

Higher Level –

Further Calculus

Question 2 Paper 2

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Further Calculus and Series, Paper II Q8

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Maximum and Minimum Problems:

These problems have the same basis as the Max/Min problems that you find in Question 6 and 7 of Paper 1. But they are more practical based and slightly more difficult. Also you will generally be given a diagram to help you visualise so make the most of that.

Remember from Differentiation:

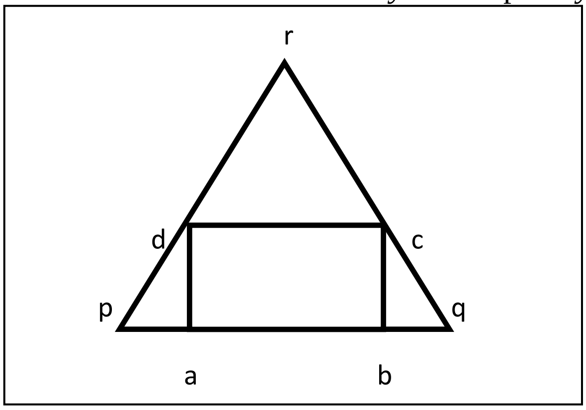
$\frac{d^2y}{dx^2} < 0$ at a **Maximum** Point

$\frac{d^2y}{dx^2} < 0$ at a **Maximum** Point

Example: Max and Min Problems Part (i)

Most questions have a part (i) to them which offers you the key doing this question. Which is expressing everything in terms of one variable.

Leaving Cert 2008, Q8 b (i) pqr is an equilateral triangle of side 6 cm. *abcd* is a rectangle inscribed in the triangle as shown. $ab = x$ cm and $bc = y$ cm. Express y in terms of x.



pq|=6

|ab|=x

As we can see from the diagram |ab| is in |pq|

|pa|=|bq|

2|bq|=6-x {This is the line pq without |ab|}

$|bq|=3-\frac{1}{2}x$

$\angle rqp = \angle cqb = 60$ {Equilateral triangle}

$\text{Tan}(\angle cqb)=\text{Tan}60=\sqrt{3}$

$\text{Tan}=\frac{\text{opp}}{\text{adj}}$ {Table Book, p. 16}

$\text{Tan}(\angle cqb)=\frac{|bc|}{bq}=\frac{y}{3-\frac{1}{2}x}=\sqrt{3}$

$y=\sqrt{3}(3-\frac{1}{2}x)$

Leaving Cert 2008, Q8 b (ii) Find the maximum possible area of *abcd*. (Continued on from the previous question)

Area of abcd = xy= x($y=(x)\sqrt{3}(3-\frac{1}{2}x)$

$$A=3x\sqrt{3}-\frac{\sqrt{3}}{2}x^2$$

$$\frac{dA}{dx}=3\sqrt{3}-\sqrt{3}x=0 \qquad \{\text{To find max, } \frac{\text{dA}}{\text{dx}}=0\}$$

$$3\sqrt{3}=\sqrt{3}x \text{ ==> } x=3$$

$$\frac{d^2A}{dx^2}=-\sqrt{3}<0 \text{==>Max}$$

A= $3x\sqrt{3}-\frac{\sqrt{3}}{2}x^2=9\sqrt{3}-\frac{9\sqrt{3}}{2}=\frac{9\sqrt{3}}{2}cm^2$

WARNING! You'll notice [redacted]

[redacted]

[redacted]

[redacted]

$$\int udv = uv - \int vdu$$

[redacted]

[redacted]

[redacted]

1. Logs
2. Inverse Trig
3. Algebra
4. Trigonometry
5. Exponential Function (e^x)

You assign the letter U to the above in that order. In each sum you will have two of these and you assign U to whichever is highest rank here.

[redacted]

[redacted]

[redacted]

Select U=x, as x is Algebra and it is higher than exponential in the order

[redacted]

[redacted]

[redacted]

[redacted]

WARNING! This is the most important step here. Watch out for this or you'll miss it

Don't forget your '+C'

[redacted]

[redacted]

[redacted]

This requires integration by parts again

[redacted]

Notice that this is the exact same as your question. This is how you know how to stop

Manipulate here: Just bring the last term across to the other side.

[redacted]

$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|$=R

R<1 ==> The series is convergent

R>1 ==> The series is divergent

R=1 ==> The test is inconclusive

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

Note: There is a button on your calculator for factorials as well. Become familiar with it. (n!)

[redacted]

[redacted]

[redacted]

[redacted]

$\frac{0}{z} = 0 \quad As\ 0 < 1, convergent$

$\frac{z}{0} = \infty \quad \text{As } \infty > 1, divergent$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

WARNING! Pay careful attention to the power of 2 here when you increase from n to n+1

It is 3(n+1)+1=3n+3+1

Not 3n+1+1

[redacted]

[redacted]

[redacted]

[redacted]

Maclaurin Series

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots\dots + \frac{f^{(n)}(0)}{n!}x^n + \dots..$$

[redacted]

[redacted]

Note that $\frac{1}{1+x}$ is converted to $(1+x)^{-1}$ for the rest of the terms and you apply the Chain Rule.

[redacted]

Note that it begins to repeat itself

[redacted]

[redacted]

[redacted]

Note the similarities between $\cos x$ *and* $\sin x$

[redacted]

WARNING! You can be asked to find $\sin^2 x$ or $\cos^2 x$, once you have sinx or cosx. Simply do this by applying the formula on p.14 Table Book. And letting x=2A

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

WARNING! It is very easy to sub in x=$\frac{11}{10}$ if you don't look closely. Be careful of this with every approximation.

[redacted]

[redacted]

This series only converges for $-1 < x < 1$. You need it to converge and therefore whatever you sub in for x must fit this. If not manipulate it so it will.

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

A good check step is to check if $\sqrt{17}$ on your calculator is close to your final answer. If they are then you should be right!

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

$$\frac{1}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

[redacted]

[redacted]

[redacted]

[redacted]

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]