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Revision Notes for Leaving Cert 2011



Maths
Leaving Cert
Probability and Statistics
Question 6,7 Paper 2

By Cillian Fahy
and Darron Higgins

Pape

r II Q.6 and 7

Discrete Mathematics, or (Probability & Statistics)

Most students tackle at least one of these questions if not both. It is very difficult to avoid this area as it leaves you with no choice at all in paper II. Probability causes some students a great amount of difficulty and it is important to thoroughly think through the question before committing to an answer. It is very easy to muddled and to jump to incorrect conclusions.

This area is broken up into the following areas

1. Permutations and Combinations
2. Probability
3. Difference Equations
4. Statistics

These questions have followed a similar pattern over the past years. Question 6(b) traditionally deals with Difference equations and Question 6(c) and 7(b) usually deals with Probability. Question 7(c) involves Statistics and the part (a)'s are made up of Permutations and Combinations. However, although this has been the pattern for the last number of years there is no guarantee that it will be the same this year. You should know something about all of these topics instead of avoiding one.

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1. Permutations and Combinations

Definition

Permutation: Must be in a set order (permutation locks, eg brief cases, bike locks)

Combination: Can be in any order.

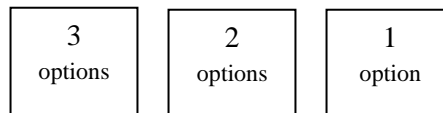
1.1 Permutations deal with the number of ways you can arrange people or things.

E.g. How many ways can 3 people sit in 3 chairs?

Ans. Lets call the people A B and C. Therefore, we can have the following ways

ABC
ACB
BAC
BCA
CAB
CBA

Therefore there are 6 ways for 3 people to sit on 3 chairs.
However, it is easier to look at permutations with boxes.



Giving $3 \times 2 \times 1 = 6$ ways

1.2 Factorial Notation:

$$3 \times 2 \times 1 = 3!$$

$$5 \times 4 \times 3 \times 2 \times 1 = 5!$$

$$\therefore n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$$

1.3 After this all the questions are basically the same, although constraints can be added. Always start with the constraint.

E.g. How many ways can the letters PROUD be arranged if a vowel must go first?

Ans. $2 \times 4 \times 3 \times 2 \times 1 = 48$ ways

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Note: If we arrange n objects of which p of one type are alike and q of another are alike is given by

$$\frac{n!}{p! \times q!}$$

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■ $7[{}^n P_3] = 6[{}^{n+1} P_3]$
 $7[n \times (n-1) \times (n-2)] = 6[(n+1) \times n \times (n-1)]$
 $7[n-2] = 6[n+1]$
 $7n - 14 = 6n + 6$
 $n = 20$

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Note: can't use our calculator for this.
Need to use above formula

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Note: Remember
AND = Multiply
OR = Add

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Note: From 2.2 we know that all events must add up to give 1.

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Note: The Queen of Clubs was counted twice. Once as a Queen and then again as a Club. Therefore, we must take away one of these repetitions.

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Note: Only 51 cards left in the deck after the first was taken out and not replaced.

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Note: Parts (i) and (ii) are not asking for the probability of an event. Remember don't rush into the question.

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$$u_n = l\alpha^n + m\beta^n$$

$$\Rightarrow u_{n+1} = l\alpha^{n+1} + m\beta^{n+1} = \alpha l\alpha^n + \beta m\beta^n$$

$$\Rightarrow u_{n+2} = l\alpha^{n+2} + m\beta^{n+2} = \alpha^2 l\alpha^n + \beta^2 m\beta^n$$

[REDACTED]

[REDACTED]

[REDACTED]

$$\begin{aligned} pu_{n+2} + qu_{n+1} + ru_n &= p(\alpha^2 l\alpha^n + \beta^2 l\beta^n) + q(\alpha l\alpha^n + \beta l\beta^n) + r(l\alpha^n + l\beta^n) \\ &= p\alpha^2 l\alpha^n + p\beta^2 l\beta^n + q\alpha l\alpha^n + q\beta l\beta^n + r l\alpha^n + r l\beta^n \\ &= l\alpha^n (p\alpha^2 + q\alpha + r) + m\beta^n (p\beta^2 + q\beta + r) \\ &= l\alpha^n (0) + m\beta^n (0) \\ &= 0 \end{aligned}$$

$$\therefore pu_{n+2} + qu_{n+1} + ru_n = 0$$

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Note: If we were then asked to find a particular term, for example u_{10} , we simply substitute 10 in for the n in the solution.

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Note: Just do what you would normally do if they were numbers., i.e. for the mean add up all the terms and divide by how many there are.

For the standard deviation take your time and don't get lost among all of the terms.

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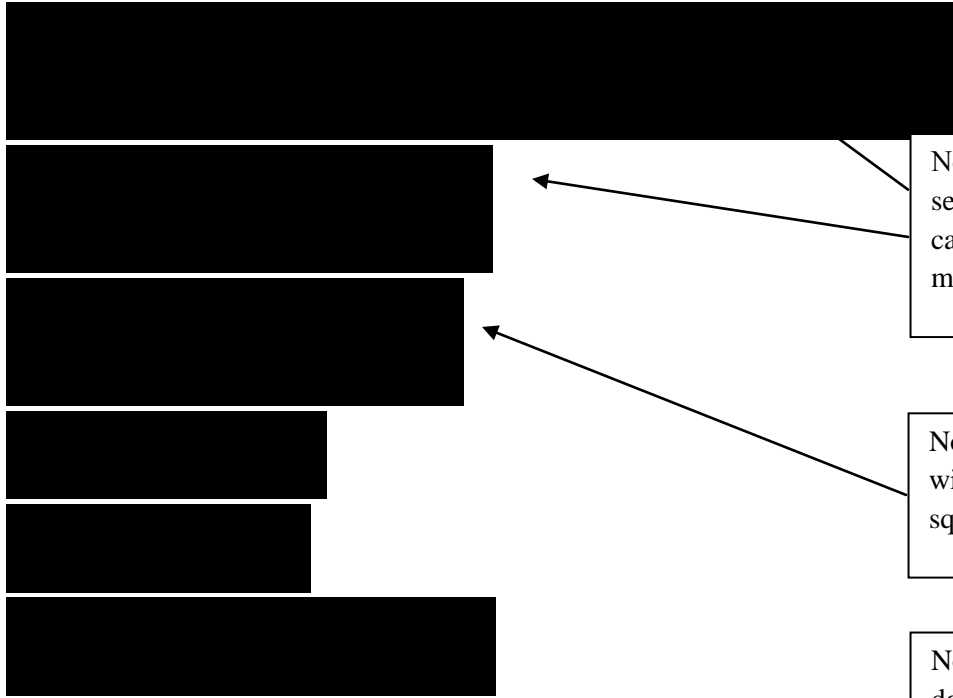
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Note: It's always a good idea to get an expanded view of the numbers that you have to deal with.

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Note: It might be very long but it is just the same as any standard deviation question.



Note: It's important to spot that the sequence of numbers is repeated and can be written on its own when multiplied by 2.

Note: From Sequences and Series you will spot that this is the sum of the squares of the first n natural numbers.

Note: Even if you are not planning on doing the Sequences and Series question you must know the basics of this topic as it can crop up anywhere.