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Revision Notes for Leaving Cert 2011

Project - Maths

Leaving Cert

Ordinary Level

Paper 2

Strand 1 - Permutations, Combinations, Probability & Statistics

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Leaving Cert Ordinary Level Notes
Paper II

Project Maths Phase 1 Strand 1

Table of Contents

Section 1: Hints and Tips

There are now **three** sections in Paper II.
The order in which the questions are asked might change from year to year but all will be included.

Section A **Concepts and Skills** **125 Marks**

5 Questions (12 minutes per question)

These questions deal with the basic formulae and methods and are short questions

Q.1 Probability, Permutations and Combinations

Q.2 Statistics

Q.3 The Line

Q.4 The Circle

Q.5A Geometry (Proofs and Constructions)

Q.5B Geometry (Proofs and Constructions)

Section B **Contexts and Applications** **125 Marks**

These questions deal with more practical applications of the information needed for Q.1 to 5 and are longer story type questions.

2 Questions (30 minutes per question)

Q.6 Further Geometry and Trigonometry

Q.7 Further Probability and Statistics

Section C **Area and Volume (old syllabus)** **50 Marks** **1 question**

1 Question (25 minutes)

Q.8 Area and Volume

- Questions must be answered in the spaces provided. It is therefore very important that your work is neat and tidy. Make sure that your work is easy to read, i.e. do not use a faint pencil!

- Additional work can be done on the space provided at the end of the exam paper. Make sure that you label these questions clearly so that the marker knows which question you are doing.

- Worded answers should be factual, brief and to the point. Do not waffle but make sure that your answer answers the question asked.

- Think through the long questions. You will need to reread the question several times and should refer back to it as you go along.

- Attempt everything! A blank space is worth 0 marks and attempt could be worth 2,5,7,10 or even 12 marks.

Section 2: Strand 1

Permutations, Combinations, Probability and Statistics

This section covers the two short questions (probably) Q.1 and Q.2 and one long question (probably) Q.7

1. Permutations and Combinations

Definition

Permutation: Must be in a set order e.g. locks on brief cases, bicycle locks, number plates.
Combination: Can be in any order e.g. choosing people to play on a team.

1.1 Permutation

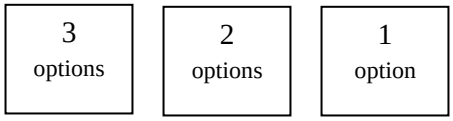
Permutations deal with the number of ways you can arrange people or things.

E.g. How many ways can 3 people sit on 3 chairs?

Ans. Lets call the people A B and C. Therefore, we can have the following ways

ABC
ACB
BAC
BCA
CAB
CBA

Therefore there are 6 ways for 3 people to sit on 3 chairs.
However, it is easier to look at permutations with boxes.



Giving $3 \times 2 \times 1 = 6$ ways

1.2 Factorial Notation:

[redacted]

1.3 Constraints

[redacted]

[redacted]

[redacted]

2 vowels so 2 options for the first box

But now still 4 letters left, so 4 options for the second box!

1.4 Permutations of *n* objects taking *r* at a time

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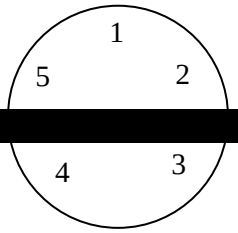

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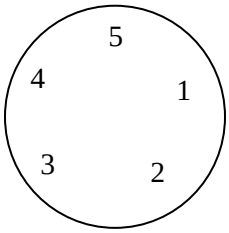

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1.5 Combinations

In combinations order is **not** important. If asked to arrange the letters ABC in a permutation we would have 3! = 6 ways. However in a combination question we would have only 1 way. ABC is the same as BCA as order is not important.
A combination therefore deals with selecting or choosing a certain amount of objects from a larger set and in how many ways this can be done. For example if there are 22 players in a squad how many different teams of 11 can be selected for this pool?

This is found using the ${}^{n}C_{r}$ button on your calculator.

$$\therefore {}^{22}C_{11} = 705432$$

Note:
This formula is in the table book, see bottom of page 20

In general terms *n* choose *r* is ${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ or $\frac{{}^{n}P_{r}}{r!}$

Note:

1. $\binom{7}{4} = \frac{7\times6\times5\times4}{1\times2\times3\times4}$ and $\binom{8}{2} = \frac{8\times7}{1\times2}$

2. $\binom{n}{r} = \binom{n}{n-r}$ e.g. $\binom{6}{2} = \binom{6}{4}$

3. $\binom{n}{n} = \binom{n}{0} = 1$

We will be able to use the calculator for most questions but should also know the above formula and the notes.

E.g.

Note: can't use our calculator for this. Need to use above formula.

1.6 Combinations from two different sets

Note: Remember

AND = Multiply

OR = Add

2. Probability

2.1 Probability

The probability of an event happening $P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

E.g. (i) $P(\text{getting a six}) = \dfrac{1}{6}$

(ii) $P(\text{getting an Ace}) = \dfrac{1}{13}$

(iii) $P(\text{getting a Spade}) = \dfrac{1}{4}$

Note: You need to know that there are 52 cards in a deck, 4 different suits (Spades, Clubs, Diamonds and Hearts), 4 of each type of card (i.e. 4 Aces etc…)

2.2. Results

2.3 Probability of an event not happening

Note: From 2.2 we know that all events must **add** up to give 1.

2.4 Sample Spaces

2.5 Rules of Probability

[redacted]

Note: The Queen of Clubs was counted twice. Once as a Queen and then again as a Club. Therefore, we must take away one of these repetitions.

[redacted]

Note: Only 51 cards left in the deck after the first was taken out and not replaced.

[redacted]

[redacted]

2.6 Probability Involving Permutations and Combinations

The more difficult questions can combine Permutations and Combinations and Probability. Remember that probability looks for the number of favourable outcomes divided by the number of possible outcomes. Use Permutations & Combinations to find the number of each outcome and then use these with the Probability rules.

E.g. In an examination a candidate is required to select any seven questions from ten.

(i) In how many ways can this be done?

(ii) How many of the selections contain the first and last questions?

Now calculate the probability that the candidate selects

(iii) both the first and second questions

(iv) at least one of the first two questions.

Note: Parts (i) and (ii) are not asking for the probability of an event. Remember don't rush into the question.

Ans. (i) $\binom{10}{7} = 120$

(ii) $\binom{8}{5} = 56$

(iii) P(both 1st and 2nd) = $\frac{56}{120} = \frac{7}{15}$

(iv) P(at least one of first two) = 1 – P(neither of first two)

Neither 1st or 2nd $= \binom{8}{7} = 8$

Therefore, P(neither 1st or 2nd) $= \frac{8}{120} = \frac{1}{15}$

Then, P(at least one of first two) = 1 – P(neither of first two)

Gives $= 1 - \frac{1}{15}$

$= \frac{14}{15}$

Q. The diagram shows two wheels.
The first wheel is divided into four equal segments numbered 1, 2, 3 and 4.
The second wheel is divided into three equal segments labelled A, B and C.

A game consists of spinning the two wheels and noting the segments that stop at the arrows. For, example the outcome shown is $(3, B)$.

(i) Write down all the possible outcomes.

(ii) What is the probability that the outcome is $(2, C)$?

(iii) What is the probability that the outcome is an odd number with the letter A?

(iv) What is the probability that the outcome includes the letter C?

Ans. (i) [redacted]

Note: time spent doing out this table properly will save you a lot of time and effort with the rest of the question. Now all we have to do is count the number of favourable options and divide by the possible outcomes (i.e. 12)

[redacted]

[redacted]

[redacted]

Q. [redacted]

[redacted]

Note: this is a very common question. It can seem a bit confusing at first but if you think it through you will find that it is not too bad. Firstly, you need to decide if it is a permutation or combination question. It helps to write out a few examples of 3 digit numbers, e.g. 243 or 562. Is 432 a different number? Yes, therefore we are looking at **arranging** the numbers as order is important (use **boxes**!)

[redacted]

[redacted]

[redacted]

[redacted]

Note: Remember to start with the constraint.

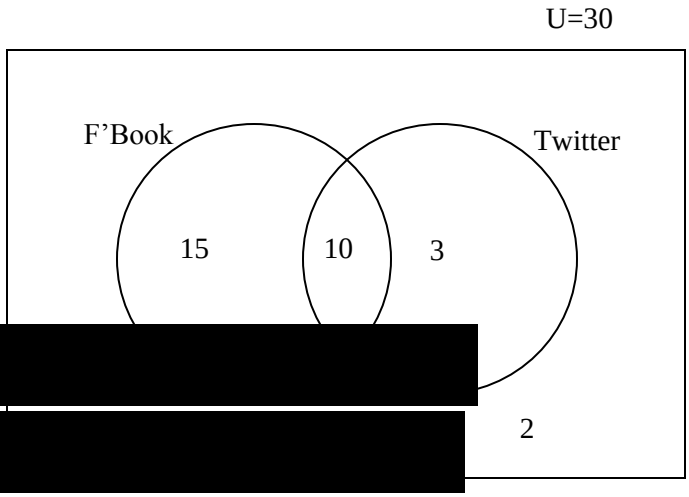
A number less than 400 must start with a 2 or a 3. Therefore, 2 options for the first box.

A number divisible by 5 must end in a 5 (no 0 in the question). Therefore, 1 option for the last box.

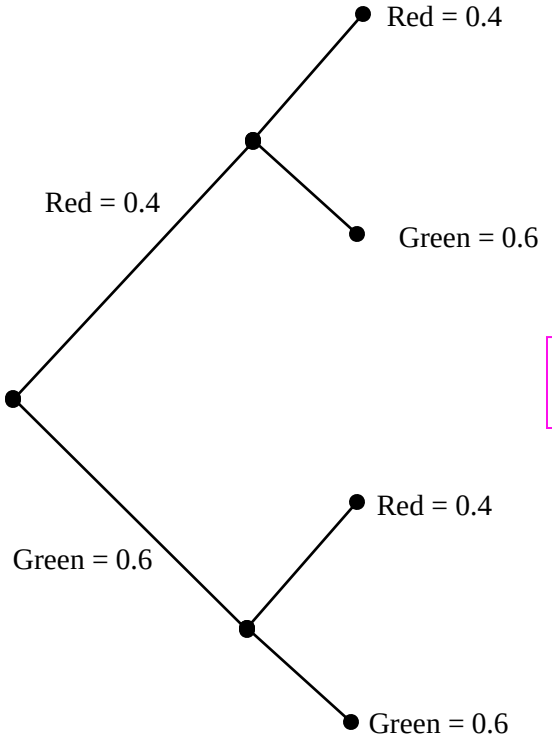
Less than 400 and divisible by 5 must start with a 2 or 3 and end with a 5. Therefore, 2 options for the first box and 1 for the last.

2.7 Venn diagrams and Probability

Note: When two sets have no elements in common then they are said to be **mutually exclusive**, i.e. cannot happen at the same time.



2.8 Tree diagrams



Note: We know that P(Green) $= 1 - 0.4 = 0.6$

Now we just need to read the answers from our tree diagram, remember AND is multiply and OR is add.

(i) P(stopped at both) = P(Red) AND P(Red)
$= 0.4 \times 0.4 = 0.16$

(ii) P(Stopped once)
=P(Red) AND P(Green) $= 0.4 \times 0.6 = 0.24$
OR +
=P(Green) AND P(Red) $= 0.6 \times 0.4 = 0.24$
} =0.48

(iii) P(Not stopped) =P(G) AND P(G)

$$0.6 \times 0.6 = 0.36$$

(iv) Note: Parts (i), (ii) and (iii) are all of the possible outcomes for this event. Therefore, they must add up together to give 1

0.16 + 0.48 + 0.36 = 1 Verified!

3. Statistics

3.1 Mean, Median and Mode

[redacted]

Note: the Mean, Median and Mode will always be relatively close together but do not have to be the same.

[redacted]

Mean of a Frequency distribution table

[redacted]

[redacted]

[redacted]

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

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[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

3.2 Standard deviation

[redacted]

[redacted]

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

[redacted]

Note: Normally we would have to work this out first

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]

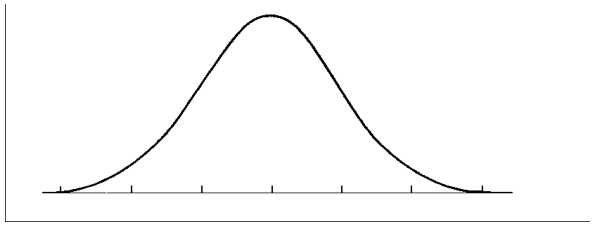
Note: You will always form the same table for a grouped frequency distribution table. However, if you are asked to find the standard deviation of a list of numbers you will use the following formula

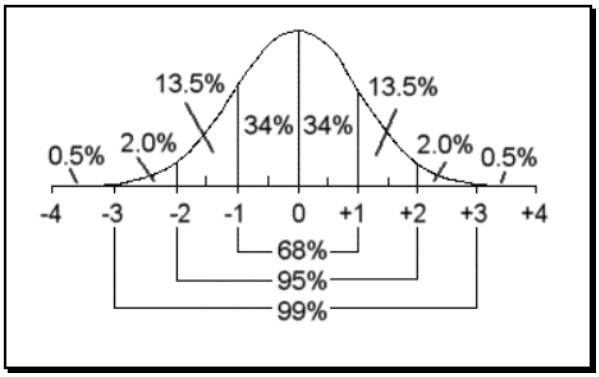
$$\sigma = \sqrt{\frac{\Sigma\left(x-\bar{x}\right)^2}{n}},$$

where n is the amount of numbers given.

Normal Distributions

[redacted]





Range

3.3 Diagrams

(a) Histograms

[redacted]

Note: Don't let this confuse you. It is always said but usually has little real impact on the question.

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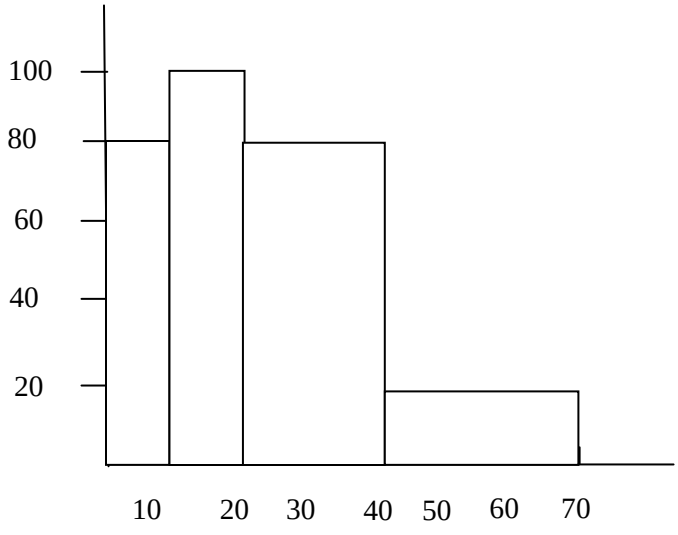
Base	1	1	2	3
Height	80	100	80	20

[redacted]



Note: Sometimes you might be given the Histogram and will be asked to fill in a frequency distribution table.

Remember the area = frequency.

(b) Cumulative Frequency Curves

[redacted]

[redacted]

[redacted]

[redacted]

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]					

[redacted]

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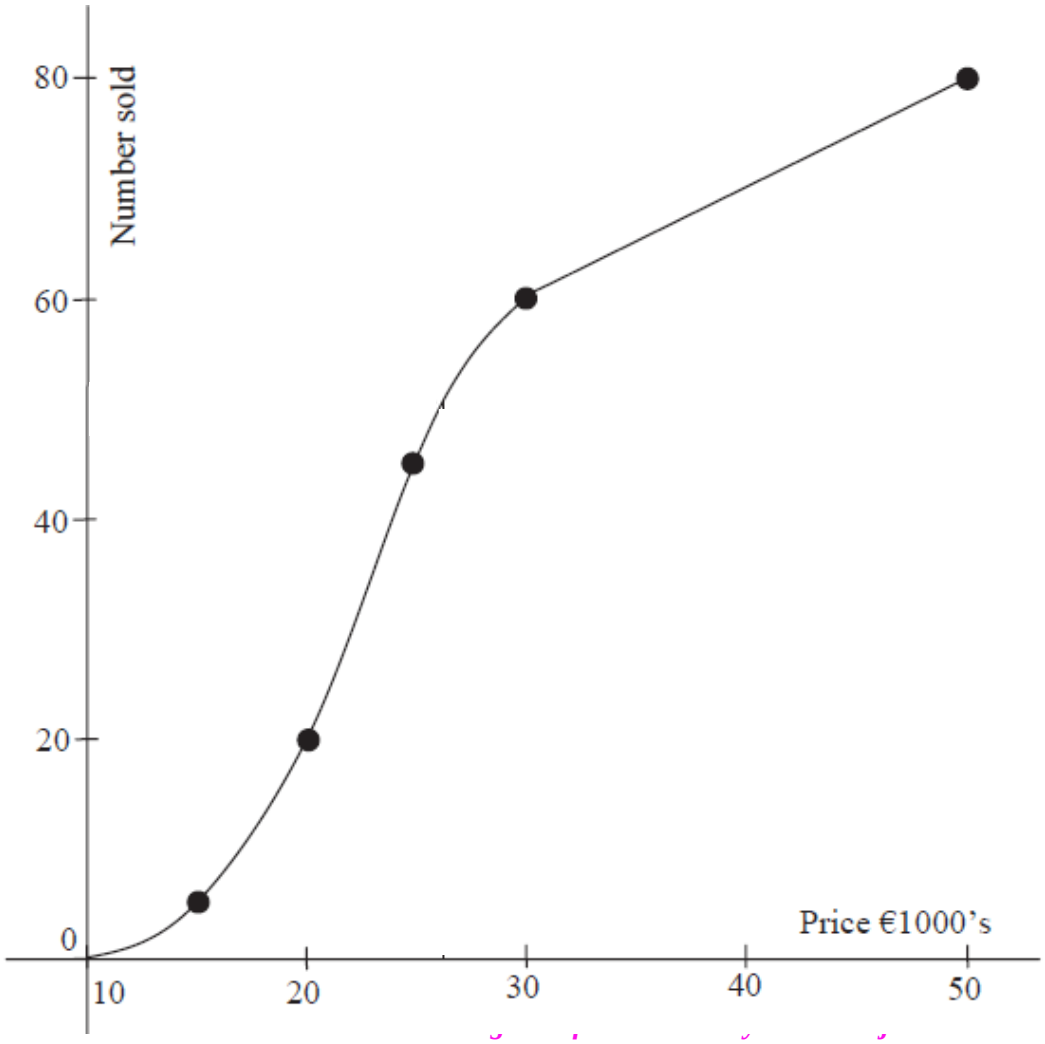
Note: We add the numbers as we go along. For example the 3rd box is 5+15+25=45 and so on and so on.

[redacted]

[redacted]



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(c) Stem and leaf diagrams

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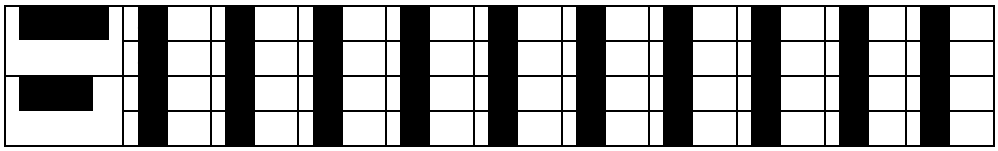

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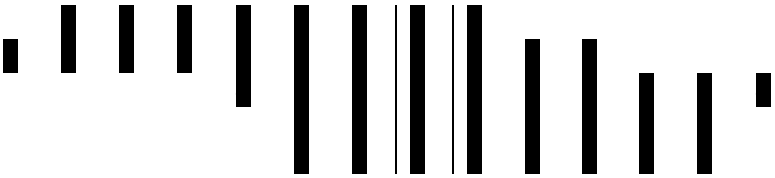

[redacted]

[redacted]

[redacted]

[redacted]

Note: There are 20 terms, therefore the median (middle) value is between the 10^{th} and 11^{th} term!

[redacted]

Note: For this last part make sure that your answer is logical and answers the question asked. Keep your answers brief and to the point i.e.**do not waffle**.

(d) Scatter diagrams.

[redacted]

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[redacted]		[redacted]	

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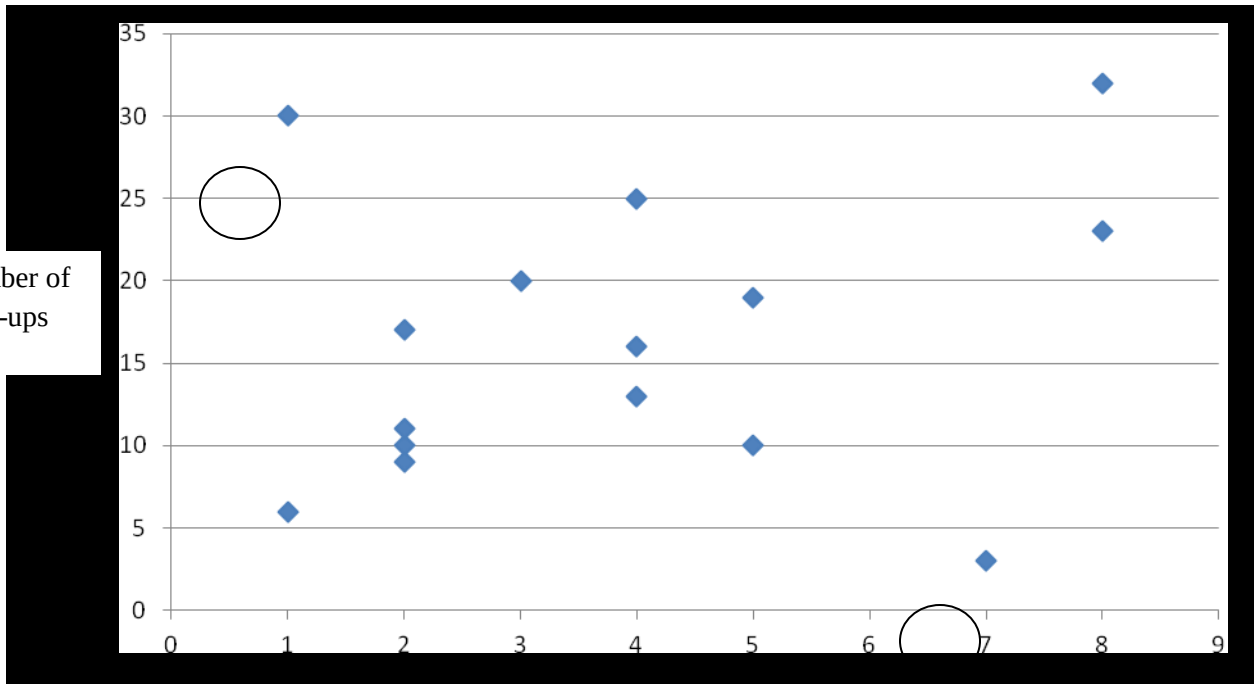

[redacted]

[redacted]

[redacted]

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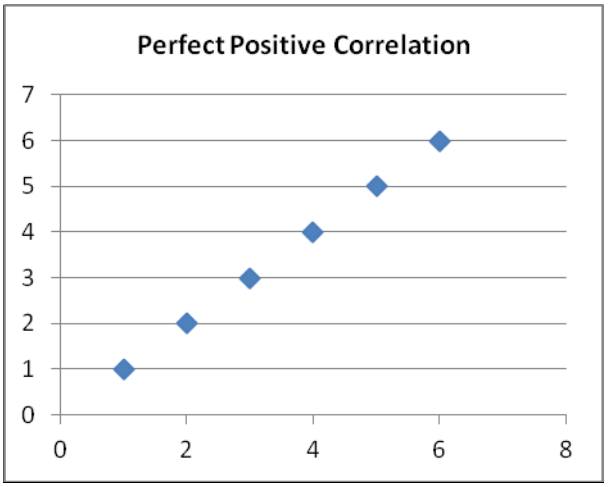
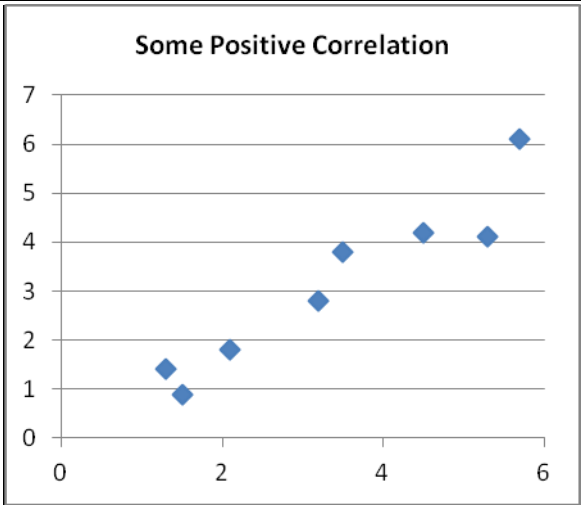
Note: For part (iii) make sure that your answer is logical and comes from the information provided in the question. Don't write too much this is a Maths exam not English!

3.4 Correlation and Causality





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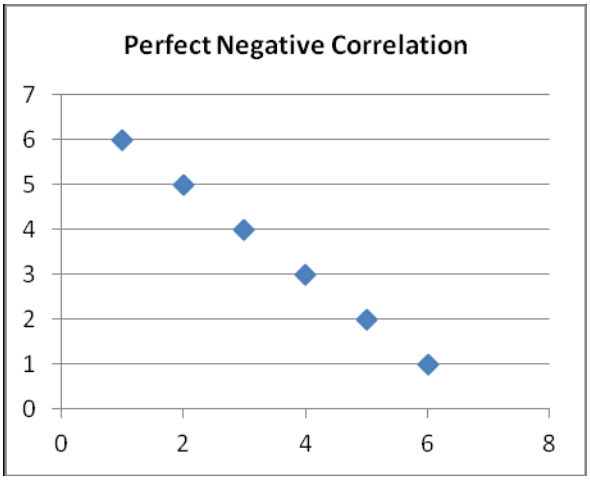

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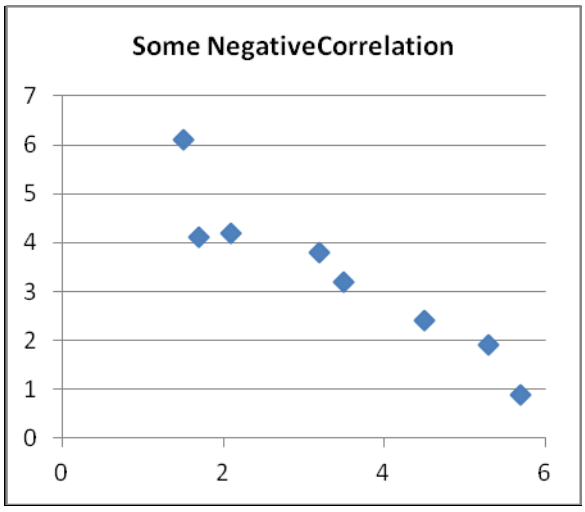

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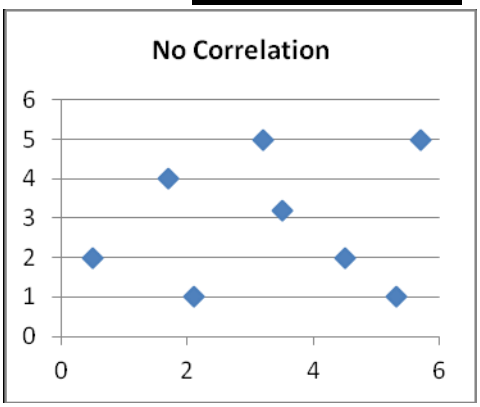

3.5 Hypothesis Testing

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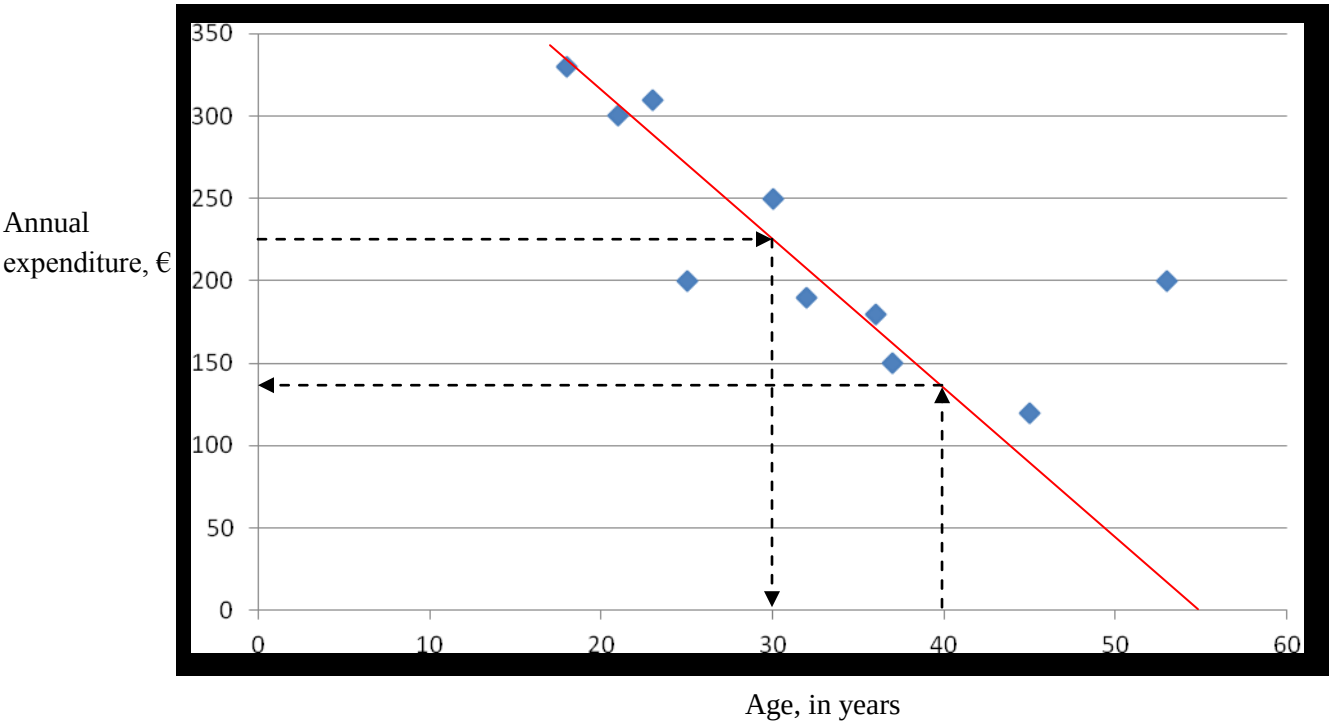

[redacted]

[redacted]

Note: The line of best fit is in red. It should have a similar number of points on each side.

[redacted]

Note: (iii) a & b are read directly from the diagram. You must show the dotted lines clearly and attempt to get as accurate a reading as possible. It is an estimate so everyone will have slightly different answers.

3.6 Shapes of distributions.

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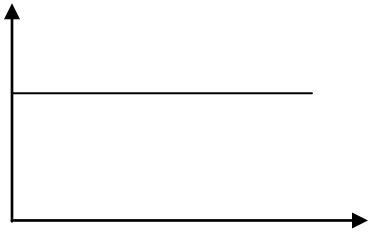

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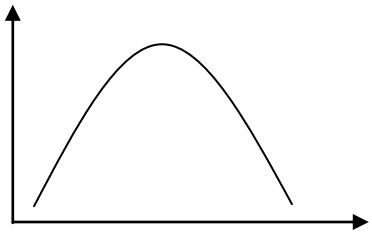

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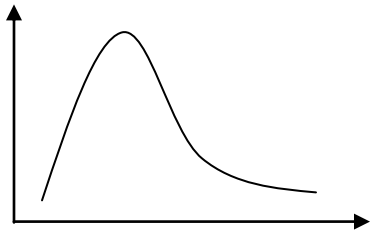

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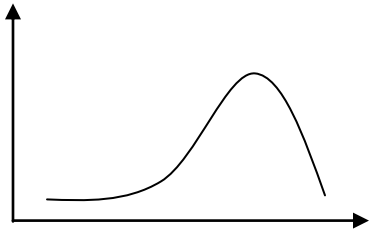

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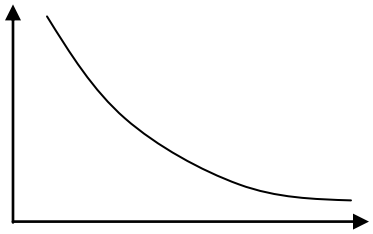

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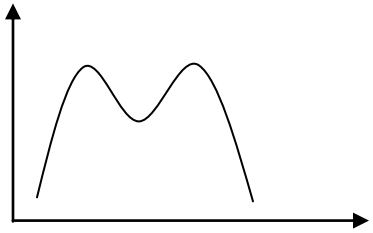

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