Project Maths

Leaving Certificate

Ordinary Level

Revision Notes

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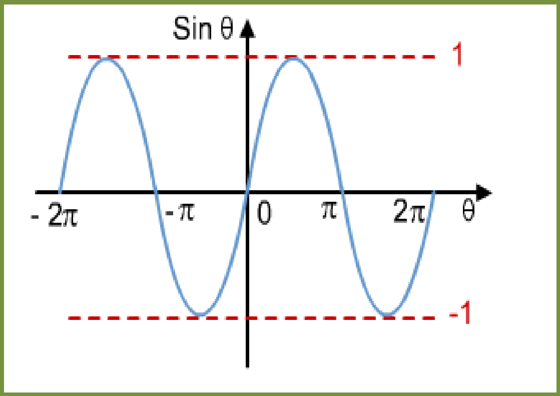

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Leaving Cert Ordinary Level Notes

Project Maths

Paper II

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Section 1: Hints and Tips

Paper II

There are **two** sections in this examination paper.

You must answer all of the questions.

The order in which the questions are asked might change from year to year but all topics will be included somewhere on the paper.

Section A **Concepts and Skills** **150 Marks**

5 Questions (approx. 12 minutes per question)

These questions deal with the basic formulae and methods.

Q.1 Probability, Permutations and Combinations

Q.2 Statistics

Q.3 The Line

Q.4 The Circle

Q.5 Area and Volume

Q.6A Geometry (Proofs and Constructions)

Q.6B Geometry (Proofs and Constructions)

Section B **Contexts and Applications** **150 Marks**

These questions deal with more practical applications of the information needed for Q.1 to 5. Read the questions carefully and slowly.

3 Questions (approx. 25 minutes per question)

Q.7 Further Geometry and Trigonometry

Q.8 Further Probability and Statistics

Q.9 Further Area and Volume

N.B. Remember;

- Questions must be answered in the spaces provided. It is therefore very important that your work is neat and tidy. Make sure that your work is easy to read, i.e. do not use a faint pencil, or a very thick fountain pen!

- Additional work can be done on the space provided at the end of the exam paper. Make sure that you label these questions clearly so that the marker knows which question you are doing.

- Worded answers should be factual, brief and to the point. Do not waffle but make sure that your answer answers the question asked.

- Think through the long questions. You will need to re-read the question several times and should refer back to it as you go along.

- Attempt everything! A blank space is worth 0 marks and attempt could be worth 2,5,7,10 or even 12 marks.

- Plan your timing carefully. Do not waste time on a question that is not going anywhere move onto the next question and come back to it at the end of the exam.

- Don't rub out or tipex out your work. The examiner will correct all work on your exam paper and you will receive the best make even if you have attempted the question more than once.

- Make sure that you know where all of the relevant formula are in the table book. You do not want to waste time in the exam looking for a formula. Remember the table book is also used for Physics, Chemistry, Economics as well as in 3rd level. It is full of stuff that you do not need for this exam.

- Make sure that you are familiar with your calculator. Do not buy a new one for the exam, different models can use different notation on the buttons or will put the buttons in a different order. Again you do not want to waste time looking for a function on your calculator.

- Use your calculator. Do not do calculations in your head (unless they are very easy). However, do not believe everything that your calculator tells you! It is very easy to hit the wrong button by mistake. Make sure that the answer makes sense to the question.

- Stay focused in the exam. Do not panic! In Maths you can always figure out an answer just take your time and read the questions carefully.

Section 2: The Line

2.1 The Line

In the past you had to learn all of the formulae required for this question. Luckily they are now given in the table book (page 18) that will be supplied on the day of your exam. However, you must familiarise yourself with them as they can be slightly different to what have in your text book. Also there are a few methods that must be learnt.

The main difference between Project Maths questions and the previous syllabus is that they try to make the questions a little less formal and a bit more practical. Therefore, you will most probably be presented with a story involving ski slopes or a car on a hill. However, after that it will boil back down to matching the correct formula with this question and the information given.

You must know the following:

- The slope of a line can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$ from your table book but you should also know that $slope = \frac{rise}{run}$.
- That slopes of **parallel lines** are equal.
- That slopes of **perpendicular lines** multiply together to give -1
  If the line L is perpendicular to M, then

$$\text{Slope of L} \times \text{Slope of M} = -1$$

Note: If you know that L and M are perpendicular and have the slope of L you can quickly find the perpendicular slope by <u>turning it upside down and changing its sign</u>, e.g. if slope of $L = \frac{2}{3}$ then a perpendicular line would have slope $-\frac{3}{2}$

- How to determining if a point is on a line.
- How to plot lines.
- How to find the point of intersection between two lines.
- How to use Translations.
- The meaning of different types of brackets,
  - $[ab] \rightarrow$ the line segment from *a* to *b*.
  - $|ab| \rightarrow$ the length of the line segment from *a* to *b*
  - $(a, b) \rightarrow$ the coordinate point where *a* is the *x*-value and *b* is the *y*-value.

Note: $|\ |$ brackets mean the measure of, or length of. Therefore, your answer must be positive. They are used as above for the length of a line $|ab| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, the area of a triangle $\text{Area } \Delta abc = \frac{1}{2}|x_1y_2 - x_2y_2|$ and can be used to ask for the measure of an angle $|\angle abc|$.

Now it is simply a case of matching up the formula with the question asked.

Q. Find the distance between the two points $(3,4)$ and $(15,9)$.

Ans. [Note: distance is the same as length]

$(3,4)$ and $(15,9)$

(x_1, y_1) (x_2, y_2)

$$\therefore \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(15-3)^2 + (9-4)^2}$$
$$= \sqrt{(12)^2 + (5)^2}$$
$$= \sqrt{144+25}$$
$$= \sqrt{169}$$
$$= 13$$

Note: Take your time.

Find the correct formula and write it in your answer book.

Substitute in the values.

Don't rush, every year a lot of students lose marks due to silly mistakes. Resist the urge to type the whole thing into your calculator.

Q. L is the line $3x - 4y + 12 = 0$

L intersects the x-axis at a and the y-axis at b.

(i) Find the co-ordinates of a and the co-ordinates of b.

(ii) K is the line that passes through b and is perpendicular to L. Show L and K on a co-ordinate diagram.

(iii) Find the equation of K.

(iv) The point $(2t, 3t)$ is on the line K. Find the value of t.

Ans. (i) L: $3x - 4y + 12 = 0$

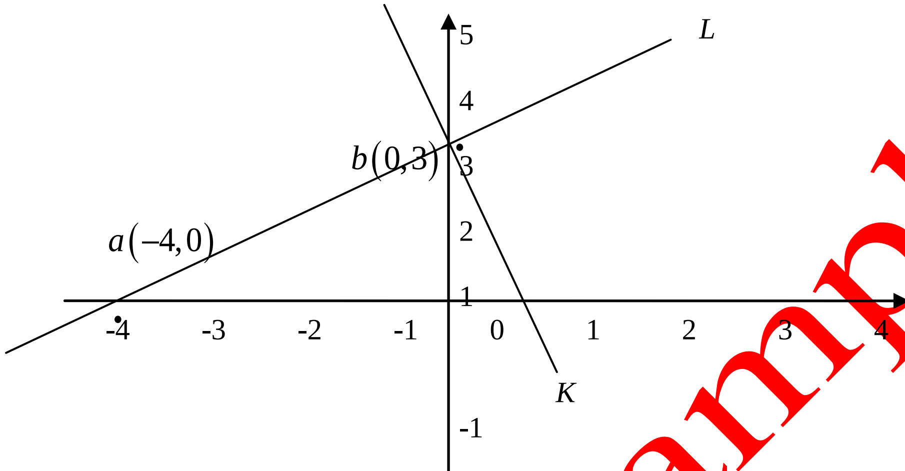
cuts the x-axis when $y = 0$ similarly cuts the y-axis when $x = 0$

$\therefore 3x - 4(0) + 12 = 0$	$\therefore 3(0) - 4y + 12 = 0$
$\Rightarrow 3x = -12$	$\Rightarrow -4y = -12$
$\Rightarrow x = -4$	$\Rightarrow y = 3$
$\therefore a(-4, 0)$	$\therefore b(0, 3)$

Note: when any line cuts the x-axis the y value must be 0. Similarly when it cuts the y-axis the x value must be 0.

Be careful to find the first point i.e. $a(-4,0)$,before moving onto the second point.

(ii)



Note: There are **two** formulas in the table book for the equation of a line

$y - y_1 = m(x - x_1)$ is used to find the equation of a line.

we need a point on the line and its slope (m).

We have $b(0,3)$ and know that the slope of K is $\perp$ to L.

$y = mx + c$ is used to find the slope of a line when given its equation.

(iii) $L: 3x - 4y + 12 = 0$

$-4y = -3x - 12$

$y = \frac{-3}{-4}x + 3$

$y = \frac{3}{4}x + 3$

Therefore, the slope of L is $\frac{3}{4}$.

Hence, the slope of K is $-\frac{4}{3}$ {Turn upside down and change the sign}

$y - y_1 = m(x - x_1)$

$\therefore y-3=-\frac{4}{3}(x-0)$

$\Rightarrow 3y-9=-4(x)$

$\Rightarrow 4x+3y-9=0$ is the equation of *K*.

(iv) K: $4x+3y-9=0$

Told that $(2t,3t)$ is on the line.

> Note: If a point is on a line it will satisfy the equation, i.e. substitute the *x* and *y* values into the equation and solve.

Therefore,

$4(2t)+3(3t)-9=0$

$8t+9t=9$

$17t=9$

$t=\frac{9}{17}$

Q. The lines $2x-y+3=0$ and $4x-y+k=0$ intersect at a point.

(i) Find, in terms of *k*, the coordinates of the point of intersection of the lines.

(ii) For what values of *k* is the point of intersection on the *y*-axis.

Ans.

> Note: This was a very tricky question and only a small percentage of students will get full marks for their answer.
> Unfortunately a lot of students didn't even start this question and you must remember to always **attempt** the question. You will get definitely get 0 if you do write anything. You might get attempt marks if you try something.

(i) $2x-y=-3$

$4x-y=-k \qquad (\times -1)$

$2x-y=-3$

$-4x+y=k$

$-2x \quad =-3+k$

$\therefore x=\frac{-3+k}{-2}$

or $x=\frac{3-k}{2}$

Putting into $2x-y=-3$ we get

$2\left(\frac{3-k}{2}\right)-y=-3$

> Note: Remember that finding the point of intersection between two lines is the same as a **simultaneous equation**. However, here we have the added confusion of a *k*. Just do what you would normally do and remember that your answer will have a *k* in it.

$$(3-k)-y=-3$$
$$3-k-y=-3$$
$$-y=-6+k$$
$y=6-k$ Therefore, point of intersection is $\left(\frac{3-k}{2}, 6-k\right)$

(ii) When a line cuts the *y*-axis the *x* value must be zero.

Therefore, $\therefore \frac{3-k}{2}=0$
$$3-k=0$$
$$k=3$$

The above example covers most of the questions that can be asked regarding the Line. Remember that the information will be wrapped up in a story of some sort (ski slopes etc..) but that everything will eventually come back to the formulas and the methods listed above. There are a few other types of questions that can be asked and they are as follows,

(i) Area of a triangle

Q. $a(-1,-2)$, $b(3,1)$, $c(0,4)$ are three points.
Find the area of the triangle *abc*.

Ans. Area $\Delta abc = \frac{1}{2}|x_1y_2 - x_2y_1|$

$a(-1,-2)$, $b(3,1)$, $c(0,4)$
↓-4 ↓-4 ↓-4
$(-1,-6)$ $(3,-3)$ $(0,0)$

Note: One point must be $(0,0)$ for this formula to work.
Move one of the points using a translation to $(0,0)$ and the others by the same translation. (in this case -4 from the *y*-value.)
Always pick the point that has to move the least.

Therefore, Area $=\frac{1}{2}|(-1)(-3)-(3)(-6)|$
$$=\frac{1}{2}|3+18|$$
$$=\frac{1}{2}|21|$$
$$=10\cdot 5 \text{ square units}$$

(ii) Proving lines are perpendicular

Q. $a(4,2)$, $b(-2,0)$, and $c(0,4)$ are three points.

Prove that $ac \perp bc$

Ans. Slope of a line is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\therefore m_{ac} = \frac{4-2}{0-4} = \frac{2}{-4} = -\frac{1}{2}$$

similarly

$$\therefore m_{bc} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$$

If $ac \perp bc$ then $m_{ac} \times m_{bc} = -1$

$$-\frac{1}{2} \times 2 = -1$$

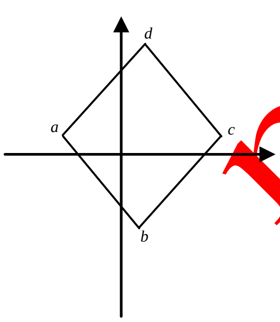
Therefore, $ac \perp bc$ as asked.

Note: You must show this last step to get full marks. You are being asked to <u>prove</u> something and your answer should reflect this by being clear and to the point.

(iii) Translations

Q. If *abcd* is a parallelogram. Find the coordinates of *d* if $a(-3,2)$, $b(1,-5)$ and $c(5,4)$

Ans. [If you are unsure draw a quick diagram, it doesn't have to be perfect but it will allow you to visualize what is going on]



From your diagram you can see that $b \rightarrow a$ the same way as $c \rightarrow d$.

$b(1,-5) \rightarrow a(-3,2)$ [Subtract 4from the *x*-value, and add 7 to the *y*-value]

$\Rightarrow c(5,4) \rightarrow d(1,11)$ [Follow the same steps as above to find *d*]