Junior Cert Maths

Free Notes

Functions and Graphing

Functions and Graphing

A function relates an input to an output

If f(x) = x^2 - x + 1 then (x) = input x^2- x + 1 = output
Here x just signifies the input it can be any letter or number

f(x) = x^2- x + 1
f(q) = q^2 - q + 1
f(A) = A^2- A + 1

If our input x = 4 then we have

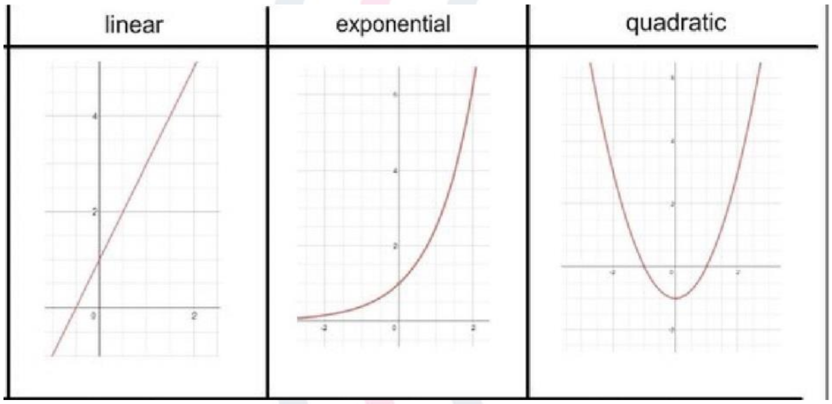
f(4) = 4^2 - 4 + 1 = 13

A function can also be expressed using y or f:x->
f(x) = x^2 - x + 1
y = x^2 - x + 1
f:x-> x^2 - x + 1

The 3 expressions above mean the exact same thing

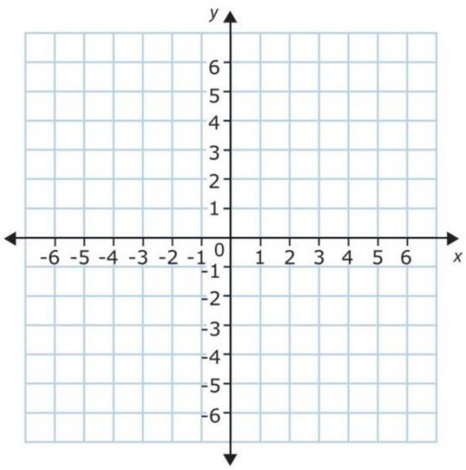
There are 3 types of basic graphs

Linear - This graph is a straight line
Exponential - This type of graph grows and shrinks very quickly
Quadratic- This graph is a parabola which means it makes a u or n shape



Co-ordinate plane

The coordinate plane looks like so



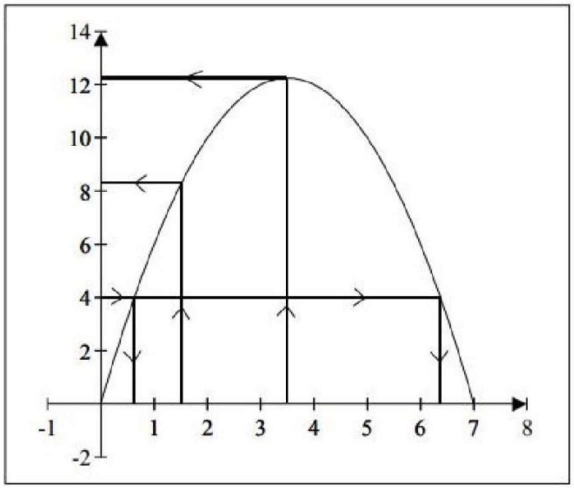
The origin is the midde point (0,0)

The x-axis is the horizontal line going across
The y-axis is the vertical line going up

A graph crosses the y- axis when x = 0
A graph crosses the x-axis when y=0

When drawing graphs we set up a table. For instance f(x) = 7x - x^2

x	7x	- x^2	f(x)	Co-ordinates
0	0	0	0	(0,0)
1	7	-1	6	(1,6)
2	14	-4	10	(2,10)
3	21	-9	12	(3,12)
4	28	-16	12	(4,12)
5	35	-25	10	(5,10)
6	42	-36	6	(6,6)
7	49	-49	0	(7,0)



Questions

1. g(x) = $\sqrt{5x-2}$, x Є N. Find g(2) in the form $a\sqrt{a}$ Є N.

Substitute in 2 for x

g(x) = $\sqrt{5x-2}$,

g(2) = $\sqrt{5(2)-2}$

g(2) = $\sqrt{8}$

g(2) = $\sqrt{4}\sqrt{2}$

g(2) = $2\sqrt{2}$ as $\sqrt{4} = 2$

2. Let f be the function f:x-> -x^2 -4x + 5, x Є R.
(i) Find the co-ordinates of the points where the graph of f(x) cuts the x-axis.

The graph cuts the x-axis when y = 0

f:x-> -x^2 -4x + 5 and
y = x^2 -4x + 5 are the exact same thing

Substitute in y = 0

$0 = -x^2 - 4x + 5$
$0 = (x - 5)(-x - 1)$
$x - 5 = 0 \qquad -x - 1 = 0$
$x = 5 \qquad\qquad x = -1$

So the co-ordinates where the graph crosses cuts the x-axis are (5,0) and (-1,0)

(ii) Solve f(x) = f(x + 1)

Substitute in x + 1 for x

$f(x + 1) = -(x + 1)^2 - 4(x + 1) + 5$
$f(x + 1) = -(x^2 + 2x + 1) - 4x - 4 + 5$
$f(x + 1) = -x^2 - 6x$

$f(x + 1) = f(x)$
$-x^2 - 6x = -x^2 - 4x + 5$

$0 = 2x + 5$
$2x = -5$

$x = \frac{5}{2}$

3.(i) Let f:x - > 5x - 4 and g be the function g:x - > 3x + 1
Using the same axes and scales, draw the graph of f and the graph of g, for $0 \leq x \leq 3$, x Є R.

3(ii) From your graphs, write down the co-ordinates of the point of intersection of the two lines.
To find our intersection point between the two lines we let f(x) = g(x)

5x - 4 = 3x + 1
5x - 3x = 1 + 4
2x = 5

x = $\frac{5}{2}$ = 2.5

Substitute x = 2.5 back into either f(x) or g(x) to find our other co-ordinate

f(x) = 3x + 1
y = 3x + 1
y = 3(2.5) + 1
y = 8.5

So our co-ordinates for intersection are (2.5, 8.5) as shown in the graph above

4. Let f be the function f:x - >1 - 3x and g be the function g:x - > 1 - x^2
(i). Find f(-2) and g(5)

f(x) = 1 - 3x
f(-2) = 1 - 3(-2)
f(-2) = 7

g(x) = 1 - x^2
g(5) = 1 - $(5)^2$
g(5) = -24

4(ii) Express f(x+1) in the form ax + b and b Є Z

Substitute in x+1 for x

f(x) = 1 -3x
f(x + 1) = 1 -3(x + 1)
f(x + 1) = 1 - 3x - 3
f(x + 1) = -2 - 3x

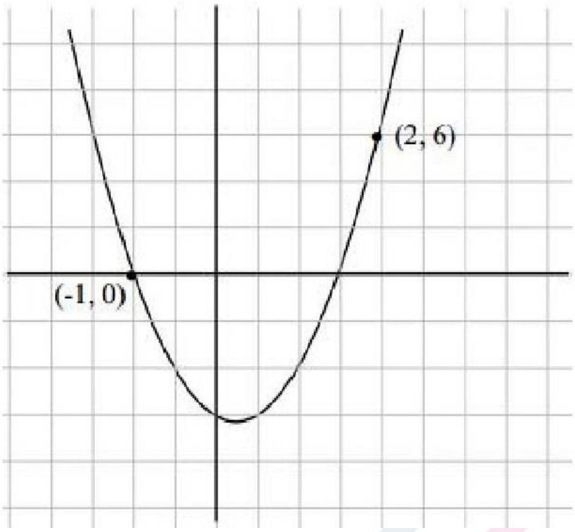
4(iii)Solve for x: f(x+1) = f(-2) + g(5)

From the previous two questions we have

-2 -3x = 7 - 24

-2 - 3x = -17
-3x = -15
x = 5

5. Let f be the function f:x -> $4x^2$ + bx + c, x ∈ R and b, c ∈ Z. The points (2, 6) and (-1, 0) lie on the graph of f, as shown in the diagram



(i) Find the value of b and the value of c

(-1,0) and (2,6) are both on the curve so they satisfy the equation of the curve

x = -1 y = 0 f:x - > $4x^2$ + bx + c

y = $4x^2$ + bx + c
0 = 4 $(-1)^2$ + b (-1) + c
0 = 4 - b + c

x = 2 y = 6 f:x - > $4x^2$ + bx + c

y = $4x^2$ + bx + c
6 = 4 $(2)^2$ + b(2) + c
0 = 16 + 2b + c - 6
0 = 10 + 2b + c

We can solve by simultaneous equations
We want to get rid of our b values

0 = 4 - b + c   Multiply by (2)
0 = 8 - 2b + 2c

0 = 10 + 2b + c
0 = 8 - 2b + 2c
0 = 18 + 0b + 3c

-18 = 3c
-6 = c

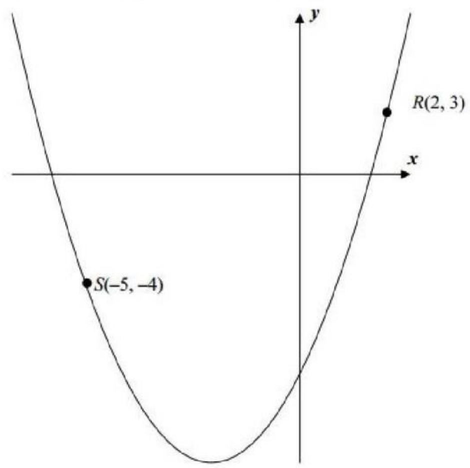
Substitute -6 =c back into 0 = 4 - b + c to find b
0 = 4 - b + c
0 = 4 - b - 6
0 = -2 -b
b = -2

(ii) Solve f(x) = -6

From our previous answers we now have the equation $4x^2$-2x -6
f(x) = -6
-6 = $4x^2$-2x -6
$4x^2$-2x = 0
2x(2x - 1) = 0
2x = 0   2x - 1 = 0
x = 0   2x = 1
                x = $\frac{1}{2}$

x = 0 and x = $\frac{1}{2}$

6. Part of the graph of the function y = x^2 + ax + b where a, b $\in$ Z is shown below



The points R(2,3) and S(-5,-4) are on the curve
(i) Use the given points to form two equations in a and b

R and S are both on the curve so they satisfy the equation of the curve

R(2,3) x = 2 y = 3
S(-5,-4) x = -5 y = -4

Substitute these values into our y = x^2 + ax + b

3 = 2^2 + a(2) + b
3 = 4 + 2a + b
0 = 1 + 2a + b

-4 = -5^2 + a(-5) + b
-4 = 25 - 5a + b
0 = 29 - 5a + b

(ii) Solve your equations to find the value of a and the value of b

Solve by simultaneous equations

We want to get rid of our b values
0 = 29 - 5a + b Multiply by (-1)
0 = -29 + 5a - b

0 = 1 + 2a + b
0 = -29 + 5a – b
0 = -28 + 7a + 0b

28 = 7a
4 = a

Substitute a = 4 back into 0 = 1 + 2a + b to find b

0 = 1 + 2a + b
0 = 1 + 2(4) + b
0 = 9 + b
b = -9

(iii) Write down the co-ordinates of the point where the curve crosses the y-axis

The curve crosses the y-axis when x = 0
Taking a = 4 and b = -9 we get our equation to be y = x^2 + 4x -9

Substitute x = 0 into our equation
y = 0^2 + 4(0) -9
y = -9

So the curve crosses the y-axis at (0,-9)

Find the points where the curve crosses the x-axis. Give your answers correct to one place of decimals.

The curve crosses the x-axis when y=0
Substitute in y = 0
0 = x^2 + 4x -9

We must use the quadratic formula x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a = 1
b = 4
c = -9

$$\frac{-4 \pm \sqrt{4^2 - 4(1)(-9)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{16 + 36}}{2}$$

$$\frac{-4 \pm \sqrt{52}}{2}$$

We split our equation into two parts

$$\frac{-4+\sqrt{52}}{2} \qquad \frac{-4-\sqrt{52}}{2}$$

Using our calculator we get

x = 1.6 and -5.6

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