Junior Cert Maths

Free Notes

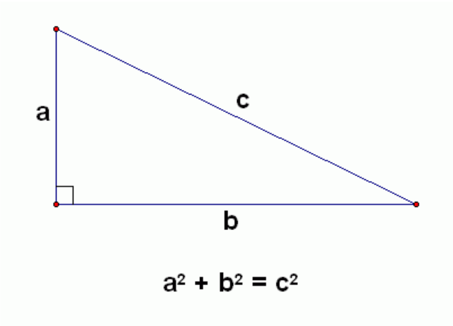
Trigonometry

Trigonometry

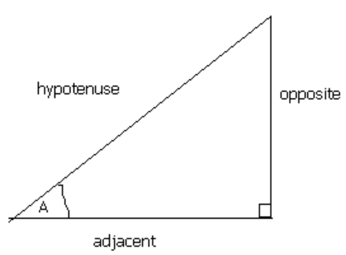
Theorem of Pythagoras

The theorem of Pythagoras says that the largest side squared is equal to the sum of the other two sides squared. The largest side is called the hypotenuse and is always opposite the 90° angle

$a^2 + b^2 = c^2$



In right-angle triangles special ratios sin, cos and tan exist between the angles and the lengths of the sides



Sin A = $\frac{\text{Opposite Side}}{\text{Hypotenuse}}$

Cos A = $\frac{\text{Adjacent Side}}{\text{Hypotenuse}}$

TanA **=** $\frac{\text{Opposite Side}}{\text{Adjacent Side}}$

If we're trying to find the angle A we use the $\sin^{-1}$ $\cos^{-1}$ $\tan^{-1}$ functions

$$A = \sin^{-1}\left(\frac{\text{Opposite Side}}{\text{Hypotenuse}}\right)$$

$$A = \cos^{-1}\left(\frac{\text{Adjacent Side}}{\text{Hypotenuse}}\right)$$

$$A = \tan^{-1}\left(\frac{\text{Opposite Side}}{\text{Adjacent Side}}\right)$$

Minutes are the same as degrees except they are expressed as fractions of 60 rather than 100

When we changes degrees to minutes or vice versa we only change the decimal part

To change degrees into minutes multiply the decimal part of degrees by $\frac{60}{100}$

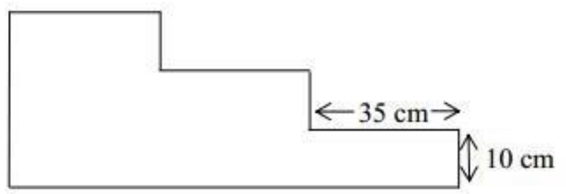
90.5° $\quad 0.5 \times \frac{60}{100} = 0.3 \quad$ 90.3 minutes

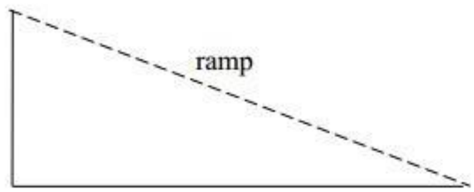
45.25° $\quad 0.25 \times \frac{60}{100} = 0.15 \quad$ 45.25 minutes

60.75° $\quad 0.75 \times \frac{60}{100} = 0.45 \quad$ 60.45 minutes

Questions

1. A homeowner wishes to replace the three identical steps leading to her front door with a ramp. Each step is 10 cm high and 35 cm long. Find the length of the ramp. Give your answer correct to one decimal place.





Height of ramp = 3 × 10 = 30

Length of Ramp = 3 × 35 = 105

We can use the theorem of Pythagoras as our ramp is Right-Angled Triangle

$a^2 + b^2 = c^2$

$30^2 + 105^2 = c^2$

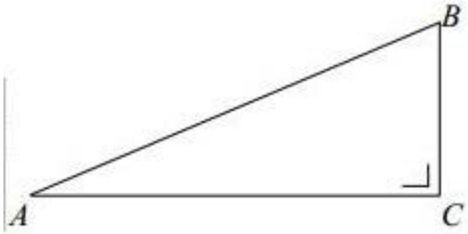
900 + 11025 = c^2

11925 = c^2

c = 109.2cm

2. In the triangle ABC, |AB| = 2 and |BC| = 1.

(a) Find |AC|, giving your answer in surd form



Use the theorem of Pythagoras

$|AB|^2 = |BC|^2 + |AC|^2$

$2^2 = 1^2 + |AC|^2$

$4 = 1 + |AC|^2$

$3 = |AC|^2$

$\sqrt{3} = |AC|$

(ii) Write Cos ∠BAC and hence find ∠BAC .

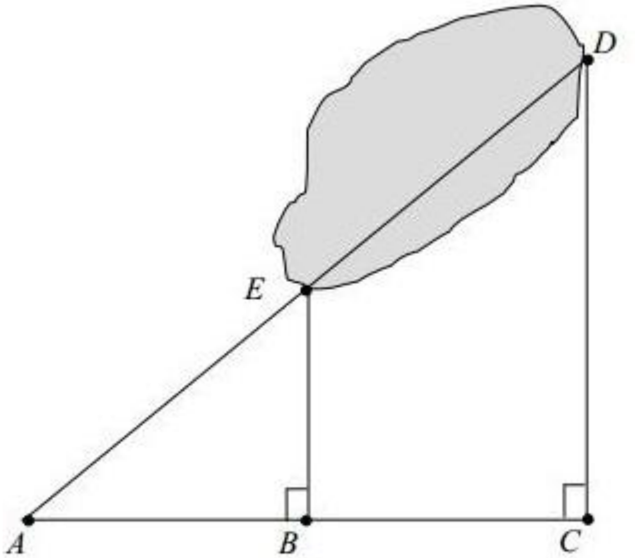
$\text{Cos } A = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$

$\text{Cos}\angle BAC = \frac{|AC|}{|AB|}$

$\text{Cos}\angle BAC = \frac{\sqrt{3}}{2}$

3. Three paths, [AE], [BE] and [CD], have been constructed to provide access to a lake from a road AC as shown in the diagram. The lengths of the paths from the road to the lake are as follows:

|AE| = 120 m |BE| = 80 m |CD| = 200 m.



(i) Explain how these measurements can be used to find |ED|.

We can use Sin A = $\frac{\text{Opposite Side}}{\text{Hypotenuse}}$

where A is the angle ∠EAB Opposite side = |BE| and Hypotenuse = |AE|

Using the angle ∠EAB we can use Sin A = $\frac{\text{Opposite Side}}{\text{Hypotenuse}}$ again where

A = ∠EAB   Opposite Side = |CD| Hypotenuse = |AD|

By doing this we can find |AD|

|ED| can then be found by subtraction

|ED| = |AD| - |AE|

(ii) Find |ED|

Sin A = $\frac{80}{120} = \frac{2}{3}$

$\frac{2}{3} = \frac{200}{|AD|}$

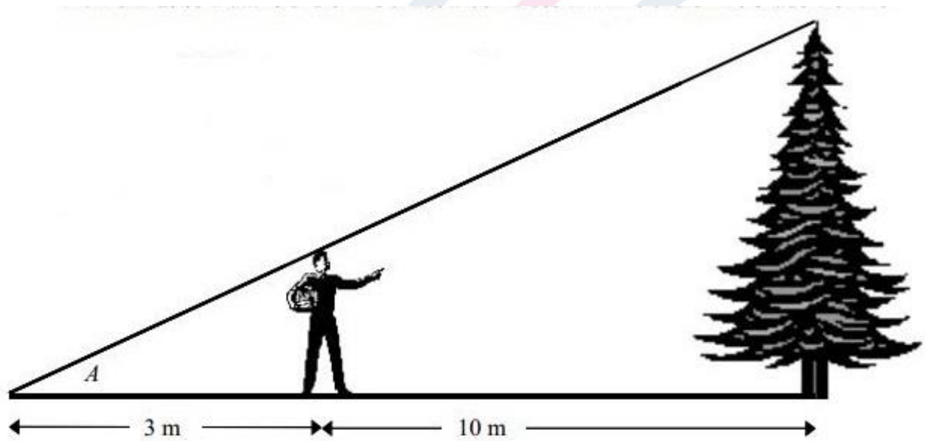
|AD| = 300

|ED| = |AD| - |AE|

|ED| = 300 – 120

|ED| = 180

4. Some students wish to estimate the height of a tree standing on level ground. One of them stands so that the end of his shadow coincides with the end of the shadow of the tree, as shown in the diagram. This student is 1·6 m tall. His friend then measures the distances shown in the diagram. A is the angle of elevation of the sun.



(i) Find A, correct to the nearest degree.

TanA **=** $\frac{\text{Opposite Side}}{\text{Adjacent Side}}$

Use the students height and the distance the student is from the tree

$\text{TanA} = \frac{1.6}{3} = 0.5333$

$A = \text{Tan}^{-1}(0.5333) = 28.07°$

A = 28.07°

A = 28° to the nearest degree

(ii) Find the height of the tree correct to one decimal place.

$\text{TanA} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$

This time use the distance to the tree and the height of the tree

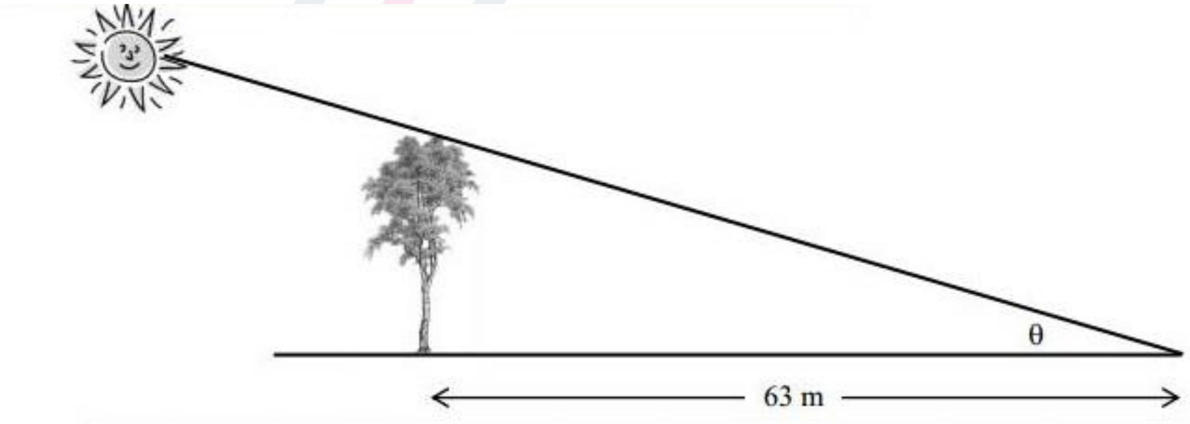
Distance to tree = 10 + 3 = 13

$0.5333 = \frac{\text{height of tree}}{13}$

6.9329 = height of tree

Height = 6.9 correct to one decimal place

5. A tree 32 m high casts a shadow 63 m long. Calculate θ , the angle of elevation of the sun. Give your answer in degrees and minutes (correct to the nearest minute).



$\text{Tan}\Theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$

$\text{Tan}\Theta = \frac{32}{63}$

$\Theta = \text{Tan}^{-1}(\frac{32}{63})$ = 26.92767785 Use your calculator for this

Θ = 26.93°

To get minutes take the 0.93 part of answer above

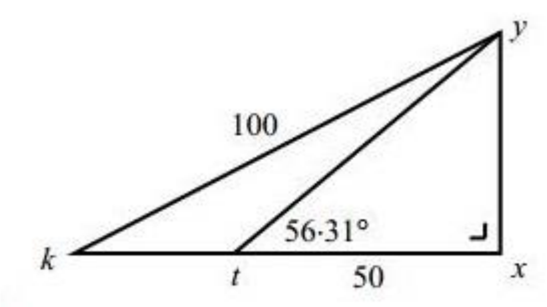
Divide 0.93 by 100 and multiply it by 60

$0.93 \times \frac{60}{100} = 55.8$

To the nearest minute 55.8 = 56

Θ = 26.56 minutes

6. A vertical mast [xy] stands on level ground. A straight wire joins y, the top of the mast, to t, a point on the ground. t is 50 m from x, the bottom of the mast.



(i) If , |∠ytx | = 56·31° find |xy| the height of the mast.

$\text{TanA} = \frac{\text{Opposite Side}}{\text{Adjacent side}}$

$\text{Tan}(56.31) = \frac{|xy|}{50}$

50(1.5) = |xy|

75 = |xy|

(ii) A second straight wire joins y to k, another point on the ground. If the length of this wire is 100 m, find ∠ykx, correct to the nearest degree

We know 75 = |xy| and 100 = |ky|

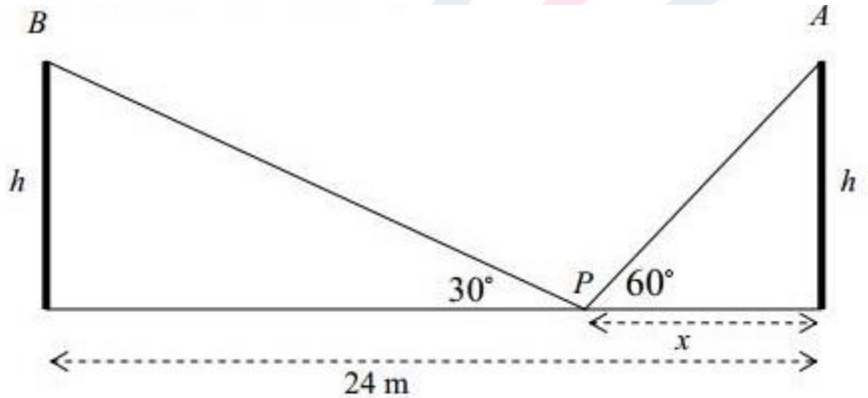
$$\text{SinA} = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$$

$$\text{Sin}(\angle ykx) = \frac{75}{100} = \frac{3}{4}$$

$$\text{Sin}^{-1}\left(\frac{3}{4}\right) = 48.59°$$

∠ykx = 49° correct to the nearest degree

7. Two vertical poles A and B, each of height h, are standing on opposite sides of a level road. They are 24 m apart. The point P, on the road directly between the two poles, is a distance x from pole A. The angle of elevation from P to the top of pole A is 60°



(i) Write h in terms of x.

$$\text{Tan60} = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

$$\text{Tan60} = \frac{h}{x}$$

$\sqrt{3} = \frac{h}{x}$

h = x $\sqrt{3}$

(ii) From P the angle of elevation to the top of pole B is 30°.
Find h, the height of the two poles

We must find x to find h

The length of the triangle on the left = 24 – x

Height of the triangle on the left = h

h = x $\sqrt{3}$

$\text{Tan}30 = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$

$\text{Tan}30 = \frac{x\sqrt{3}}{24 - x}$

$\frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{24 - x}$

$\frac{24-x}{\sqrt{3}} = x\sqrt{3}$

24 - x = $x\sqrt{3}\sqrt{3}$

24 –x = $3x$

24 = 4x

x = 6

h = x $\sqrt{3}$

h = 6$\sqrt{3}$ = 10.39

For more comprehensive Junior Cert Revision Notes Click Here…. Junior Cert Maths Notes