Junior Cert Maths

Free Notes

Cylinder and Spheres

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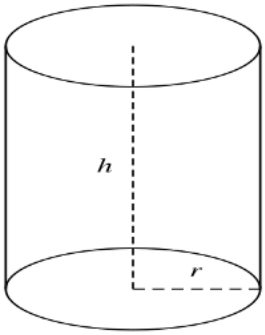
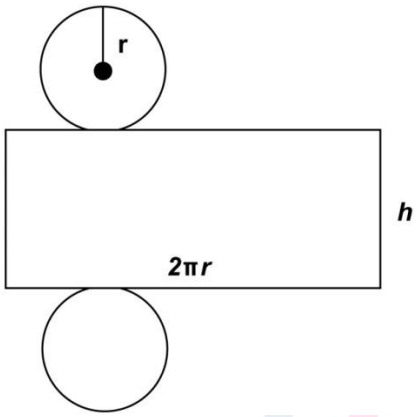
Volume of a Clinder = $\pi r^2 h$



r is the radius and h is the height or length of the cylinder

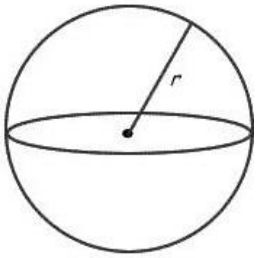
Curved surface Area of a cylinder= $2\pi rh$ (This is the area of the rectangular part)

Total Surface Area = $2\pi rh$ + $2\pi r^2$ (This is made up of its curved surface area plus its two circular ends)



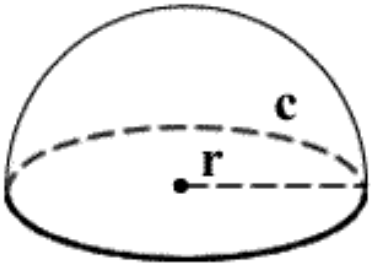
This is made up of its curved surface area plus its two circular ends

Volume of a sphere= $\frac{4}{3}$ π r^3



Surface Area of a sphere = $4\pi r^2$

Volume of a hemisphere = $\frac{2}{3}\pi r^3$



Surface area of a hemisphere = $2\pi r^2$

Questions

1. Find the total surface area of a solid hemisphere of diameter 14 cm. Give your answer correct to the nearest whole number.

Diameter = $\frac{1}{2}$ (radius)

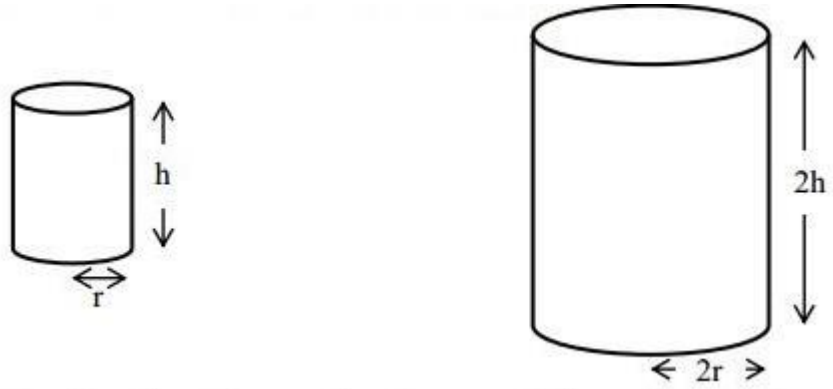
Diameter = $\frac{1}{2}$ (14) =7

Hemisphere surface area = $2\pi r^2$

Hemisphere surface area = $2(3.14)(7)^2$

Hemisphere surface area **=** 307.72

2. The dimensions of two solid cylinders are shown in the diagrams below.



(i) Calculate the ratio of the curved surface area of the smaller cylinder to the curved surface area of the larger cylinder.

Curved Surface Area = $2\pi rh$

Small Cylinder Curved Surface Area = $2\pi rh$

Small Cylinder Curved Surface Area = $2\pi(2r)(2h) = 8\pi rh$

Ratio = $2\pi rh : 8\pi rh$

Divide both sides of the ratio by $2\pi rh$

Ratio = 1 : 4

(ii) Calculate the ratio of the volume of the smaller cylinder to the volume of the larger cylinder.

Volume = $\pi r^2 h$

Small Cylinder Volume = $\pi r^2 h$

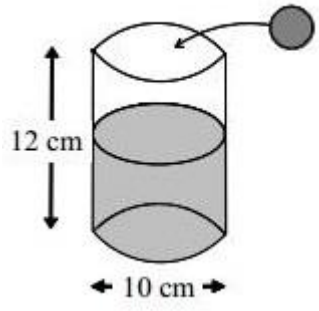
Large Cylinder Volume = $\pi(2r)^2 2h = 8\pi r^2 h$

Ratio = $\pi r^2 h : 8\pi r^2 h$

Divide both sides by $\pi r^2 h$

Ratio = 1 : 8

3. A spherical golf ball has a diameter of 4 cm.



(i) Find the volume of the golf ball in terms of π .

radius = $\frac{1}{2}$ (diameter)

radius = $\frac{1}{2}$ (4) =2

Volume of a sphere= $\frac{4}{3}$ π r^3

Volume of golf ball = $\frac{4}{3}$ π $(2)^3$

Volume of golf ball = $\frac{32\pi}{3}$

(ii) A cylindrical hole on a golf course is 10 cm in diameter and 12 cm deep. The hole is half full of water.

Calculate the volume of water in the hole, in terms of π

radius = $\frac{1}{2}$ (diameter)

radius = $\frac{1}{2}$ (10) =5

Volume of a cylinder **=** $\pi r^2 h$

Volume of cylinder **=** $\pi(5)^2(12)$

Volume of Cylinder = 300π

The hole is half full of water.

$\frac{1}{2}$ (300π) = 150 π

(iii) The golf ball is dropped into the hole. Find the rise in the level of the water, correct to two decimal places.

Volume of water + golf ball = $\frac{32\pi}{3}$ + 150 π

Volume of water + golf ball = 160.67 π

We want to find the new height of the water

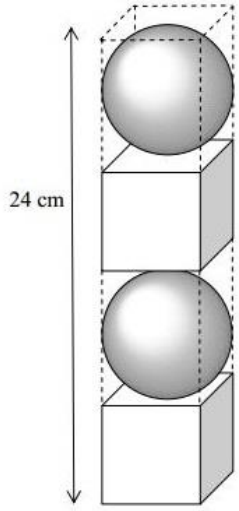
Volume of a cylinder = $\pi r^2 h$

160.67 π = $\pi(5)^2(h)$

160.67 = 25h

6.43 = h

4. An ornament is carved from a rectangular block of wood which has a square base and a height of 24 cm. The ornament consists of two identical spheres and two identical cubes as illustrated in the diagram. The diameter of each sphere is equal to the length of the side of each cube. The ornament has the same width as the original block.



(i) Find the length of a side of one of the cubes.

The two spheres and the two rectangular blocks each have the same height

So $\frac{24}{4}$ = 6cm

6cm = height of a block/Diameter of a Sphere

For a cube length = height = width

Cube length = 6cm

(ii) Find the volume of the ornament.

Volume of cube = length × height × width

Volume of cube = 6^3 = 216

Volume of a Sphere = $\frac{4}{3}\pi r^3$

Radius of the sphere is half the diameter

Radius = $\frac{1}{2}$ (6) = 3

Volume of a Sphere = $\frac{4}{3}$ (3.14) 3^3

Volume = 113.04

Total Volume = Volume of two Spheres + Volume of two Cubes

Total Volume = 216 + 216 +113.04 + 113.04

Total Volume = 658.08

(iii) In making the ornament, what percentage of the original block of wood is carved away?

Volume = length × Height × Width

Height = 24cm

It has a Square base made from the cube so

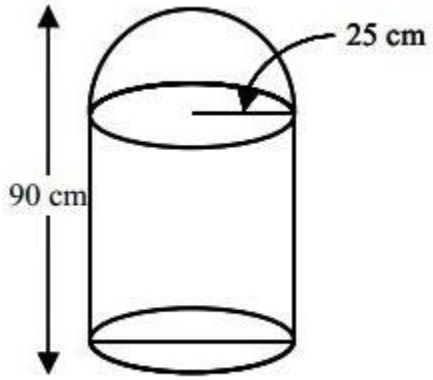
Length = 6cm

Width = 6cm

Volume =6 × 24 × 6 = 864

864 - 658.08 = 205.92

5. A hot water container is in the shape of a hemisphere on top of a cylinder as shown. The hemisphere has a radius of 25 cm and the container has a height of 90 cm. Find the internal volume of the container in litres, giving your answer correct to the nearest litre



Height of Cylinder = 90 - (radius of the hemisphere)

Height of Cylinder = 90 – 25 =65

Volume of cylinder = $\pi r^2 h$

Volume of cylinder = $(3.14)(25)^2(65)$

Volume of Cylinder = 127562.5

Volume of Hemisphere = $\frac{2}{3}\pi r^3$

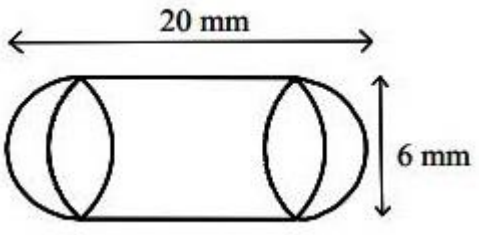
Volume of Hemisphere = $\frac{2}{3}(3.14)(25)^3$

Volume of Hemisphere = 32708.33

Total Volume = 127562.5 + 32708.33 = 160270.83

Total number of litres = 160 litres

6.. A vitamin capsule is in the shape of a cylinder with hemispherical ends. The length of the capsule is 20 mm and the diameter is 6 mm.



(i) Calculate the volume of the capsule, giving your answer correct to the nearest mm3

Volume of a cylinder $= \pi r^2 h$

Radius = $\frac{1}{2}$ (diameter)

Radius = $\frac{1}{2}$ (6) = 3

Height = 20 -(radii of the 2 hemispheres)

Height = 20 - 2(3)

Height = 20 -6 = 14

Volume = $(3.14)(3)^2(14)$

Volumeof cylinder = 395.64

Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}(3.14)(3)^3$

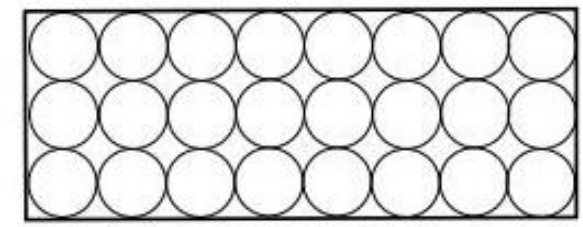
Volume = of hemisphere = 56.52

hemispheres on both ends so 2(56.52) = 113.04

Total Volume = 395.64+ 113.04 = 508.68

Total Volume = 509 correct to the nearest mm3

(ii) A course of these vitamins consists of 24 capsules. The capsules are stacked in three rows of eight in a box as shown in the diagram.



(ii) How much of the internal volume of the box is not occupied by the capsules

Volume of the box is length × height × width

Length = 8(diameter of a capsule) = 8(6) = 48

Width = 3(diameter of a capsule) = 3(6) = 18

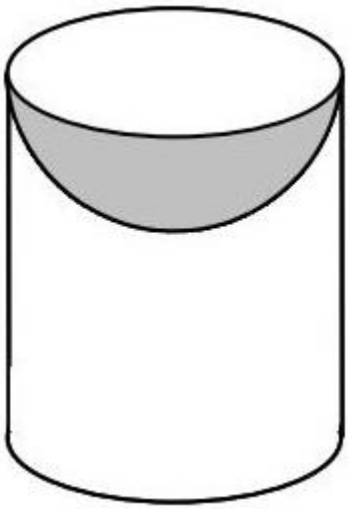
Height = 20

Volume of the box = (48)(18)(20) = 17280

Volume occupied by the capsules = 24(509) = 12216

17280 – 12216 = 5064

7.



(i) A solid metal cylinder has height 20 cm and diameter 14 cm.
Find its curved surface area in terms of π.

Radius = $\frac{1}{2}$ (diameter)

Radius = $\frac{1}{2}(14) = 7$

Curved surface Area of a cylinder = $2\pi rh$

Curved surface Area of a cylinder = $2\pi(7)(20) = 280\,\pi$

(ii) A hemisphere with diameter 14 cm is removed from the top of this cylinder, as shown. Find the total surface area of the remaining solid in terms of π

For our total surface area we have

Curved Suface Area + Circular Surface Area on the bottom + Hemisphere surface area on top

Total Surface area = $280\,\pi + \pi r^2 + 2\,\pi r^2$

Total Surface Area = $280\,\pi + \pi(7)^2 + 2\,\pi(7)^2$

Total Surface Area = $280\,\pi + 49\pi + 98\,\pi = 427\,\pi$

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