Differentiation

Maths Past Exam Questions

Higher Level

2012

Paper 1 – Section A – Q6

6. **(a)** Differentiate with respect to x:

(i) $(4x^2 - 1)^3$.

(ii) $\sin^{-1}\left(\frac{2x}{3}\right)$.

(b) **(i)** Differentiate $\sqrt{x}$ with respect to x, from first principles.

(ii) Find the equation of the tangent to the curve $y = \sqrt{x}$ at the point $(9, 3)$.

(c) Let f be the function $f : x \to 8x + \sin 4x + 4\sin 2x$, where $x \in \mathbb{R}$.

(i) Find $f'(x)$.

(ii) Express $f'(x)$ in terms of $\cos 2x$.

(iii) Prove that $f(x)$ is increasing for all values of x.

2011

Paper 1 – Section A – Q6

6. **(a)** Differentiate $\cos^2 x$ with respect to x.

(b) The equation of a curve is $y = e^{-x^2}$.

(i) Find $\dfrac{dy}{dx}$.

(ii) Find the co-ordinates of the turning point of the curve.

(iii) Determine whether this turning point is a local maximum or a local minimum.

(c) The function f is defined as $x \to \dfrac{2x}{x+1}$, where $x \in \mathbb{R} \setminus \{-1\}$.

(i) Find the equations of the asymptotes of the curve $y = f(x)$.

(ii) P and Q are distinct points on the curve $y = f(x)$.
The tangent at Q is parallel to the tangent at P.
The co-ordinates of P are (1, 1).
Find the co-ordinates of Q.

(iii) Verify that the point of intersection of the asymptotes is the midpoint of $[PQ]$.

2010

Paper 1 – Section A – Q6

6. **(a)** The equation $x^3 + x^2 - 4 = 0$ has only one real root.

Taking $x_1 = \frac{3}{2}$ as the first approximation to the root, use the Newton-Raphson method to find x_2, the second approximation.

(b) Parametric equations of a curve are:

$$x = \frac{2t-1}{t+2}, \qquad y = \frac{t}{t+2}, \qquad \text{where } t \in \mathbb{R} \setminus \{-2\}.$$

(i) Find $\frac{dy}{dx}$.

(ii) What does your answer to part **(i)** tell you about the shape of the graph?

(c) A curve is defined by the equation $x^2y^3 + 4x + 2y = 12$.

(i) Find $\frac{dy}{dx}$ in terms of x and y.

(ii) Show that the tangent to the curve at the point $(0, 6)$ is also the tangent to it at the point $(3, 0)$.

2009

Paper 1 – Section A – Q6

6. **(a)** Differentiate $\sin(3x^2 - x)$ with respect to x.

(b) **(i)** Differentiate $\sqrt{x}$ with respect to x, from first principles.

(ii) An object moves in a straight line such that its distance from a fixed point is given by $s = \sqrt{t^2 + 1}$, where s is in metres and t is in seconds.

Find the speed of the object when $t = 5$ seconds.

(c) The equation of a curve is $y = \dfrac{2}{x-3}$.

(i) Write down the equations of the asymptotes and hence sketch the curve.

(ii) Prove that no two tangents to the curve are perpendicular to each other.

2007

Paper 1 – Section A – Q6

6. **(a)** Differentiate $\frac{x^2-1}{x^2+1}$ with respect to x.

(b) **(i)** Differentiate $\frac{1}{x}$ with respect to x from first principles.

(ii) Find the equation of the tangent to $y = \frac{1}{x}$ at the point $\left(2, \frac{1}{2}\right)$.

(c) Let $f(x) = \tan^{-1}\frac{x}{2}$ and $g(x) = \tan^{-1}\frac{2}{x}$, for $x > 0$.

(i) Find $f'(x)$ and $g'(x)$.

(ii) Hence, show that $f(x) + g(x)$ is constant.

(iii) Find the value of $f(x) + g(x)$.

2006

Paper 1 – Section A – Q6

6. **(a)** Differentiate $\sqrt{x}(x+2)$ with respect to x.

(b) The equation of a curve is $y = 3x^4 - 2x^3 - 9x^2 + 8$.

(i) Show that the curve has a local maximum at the point $(0, 8)$.

(ii) Find the coordinates of the two local minimum points on the curve.

(iii) Draw a sketch of the curve.

(c) Prove by induction that $\frac{d}{dx}\left(x^n\right) = nx^{n-1},\ n \geq 1,\ n \in \mathbf{N}$.

2005

Paper 1 – Section A – Q6

6. **(a)** Differentiate with respect to x

(i) $(1+7x)^3$ **(ii)** $\sin^{-1}\left(\frac{x}{5}\right)$.

(b) Let $y = \frac{1-\cos x}{1+\cos x}$.

Show that $\frac{dy}{dx} = t + t^3$, where $t = \tan\frac{x}{2}$.

(c) The equation of a curve is $y = \frac{x}{x-1}$, where $x \neq 1$.

(i) Show that the curve has no local maximum or local minimum point.

(ii) Write down the equations of the asymptotes and hence sketch the curve.

(iii) Show that the curve is its own image under the symmetry in the point of intersection of the asymptotes.