Maths
Leaving Certificate
Ordinary Level

Past Exam Questions
Marking Scheme on
Algebra

Q9 2013 Paper One Project Maths Section B

Question 9 (50 marks)

A company has calculated that the daily cost (in euro) to produce x items is given by the production cost function $C(x) = 5x^2 + 750x + 3000$. The total daily income from the sale of x items is given by the revenue function $R(x) = 1200x$.

The company assumes that it will sell all the items it produces.

(a) The company produces 20 items in one day. Find the production cost and total income for the 20 items.

> Production cost: $C(x) = 5x^2 + 750x + 3000$
> $\Rightarrow C(20) = 5(20)^2 + 750(20) + 3000 = €20\,000$
>
> Total income: $R(x) = 1200x$
> $\Rightarrow R(20) = 1200(20) = €24\,000$

(b) Find the profit the company makes on that day.

> Profit: $R(20) - C(20) = 24000 - 20000 = €4000$

(c) Find a general expression for the profit the company makes from the production of x items.

> Profit: $P(x) = R(x) - C(x) = 1200x - (5x^2 + 750x + 3000)$
> $\Rightarrow P(x) = -5x^2 + 450x - 3000$

(d) How many of these items will the company have to produce and sell in order to make a maximum profit?

> $P(x) = -5x^2 + 450x - 3000$
> $\Rightarrow P'(x) = -10x + 450$
>
> $P'(x) = 0 \Rightarrow -10x + 450 = 0$
> $\Rightarrow x = 45$

(e) Find the maximum profit the company can make.

> $P(x) = -5x^2 + 450x - 3000$
>
> $P(45) = -5(45)^2 + 450(45) - 3000 = -10125 + 20\,250 - 3000 = €7125$

(f) The production costs on a particular day amount to €11 000. Find number of items produced on that day.

$$C(x) = 5x^2 + 750x + 3000 = 11000$$
$$\Rightarrow 5x^2 + 750x - 8000 = 0$$
$$\Rightarrow x^2 + 150x - 1600 = 0$$
$$\Rightarrow (x - 10)(x + 160)$$
$$\Rightarrow x = 10 \quad \text{or} \quad x = -160$$

Answer: 10 items

Or

$$C(x) = 5x^2 + 750x + 3000$$
Let $x = 10$
$$C(10) = 5(10)^2 + 750(10) + 3000$$
$$= 500 + 7500 + 3000$$
$$= 11000$$

The production costs amount to €11 000 when 10 items are produced.

Question 7 (50 marks)

Doctors sometimes need to work out how much medicine to give a child, based on the correct dose for an adult. There are different ways of doing this, based on the child's age, weight, height, or some other measure.

(a) One rule for working out the child's dose from the adult dose is called *Clark's rule*. It is:

$$C = \left(\frac{W}{68}\right) \times A$$

where C is the child's dose, A is the adult's dose, and W is the child's weight in kilograms.

The adult dose of a certain medicine is 125 mg per day. Calculate the correct dose for a child weighing 30 kg, using Clark's rule. Give the answer correct to the nearest 5 mg.

$$C = \left(\frac{W}{68}\right) \times A$$
$$= \left(\frac{30}{68}\right) \times 125$$
$$= 55{\cdot}147 \approx 55$$

Answer: 55 mg

(b) Another rule for working out the child's dose is called *Young's rule*. Below are three different descriptions of Young's rule, taken from the internet. In each case, write down a formula that *exactly* matches the description in words. State clearly the meaning of any letters you use in your formulae.

(i) **Young's rule:** a mathematical expression used to determine a drug dosage for children. The correct dosage is calculated by dividing the child's age by an amount equal to the child's age plus 12 and then multiplying by the usual adult dose.

Mosby's Dental Dictionary, 2nd edition.

Formula: $C = \left(\frac{Y}{Y+12}\right) \times A$

where C is child's dose, Y is the child's age in years, A is the adult's dose.

(ii) **Young's rule:** A rule for calculating the dose of medicine correct for a child by adding 12 to the child's age, dividing the sum by the child's age, then dividing the adult dose by the figure obtained.

The American Heritage Medical Dictionary

Formula: $C = \dfrac{A}{\left(\dfrac{Y+12}{Y}\right)}$ C, A and Y same meaning as (i)

(iii) **Young's rule:** the dose of a drug for a child is obtained by multiplying the adult dose by the child's age in years and dividing the result by the sum of the child's age plus 12.

Miller-Keane Encyclopedia and Dictionary of Medicine, Nursing, and Allied Health, Seventh Edition.

Formula: $C = \dfrac{A \times Y}{Y+12}$ C, A and Y same meaning as (i)

(c) Explain why the three formulae in (b) above all give the same result.

By algebraic manipulation each formula can be written in the same form $C = \dfrac{YA}{Y+12}$

(d) The adult dose of a certain medicine is 150 mg per day. According to Young's rule, what is the correct dose for a six-year old child?

$$C = \frac{YA}{Y+12} = \frac{6 \times 150}{6+12} = 50 \text{ mg per day}$$

(e) Young's rule results in a certain child being given one fifth of the adult dose of a medicine. How old is this child?

$$C = \frac{YA}{Y+12} \Rightarrow \frac{A}{5} = \frac{YA}{Y+12} \Rightarrow Y+12 = 5Y \Rightarrow 4Y = 12 \Rightarrow Y = 3$$
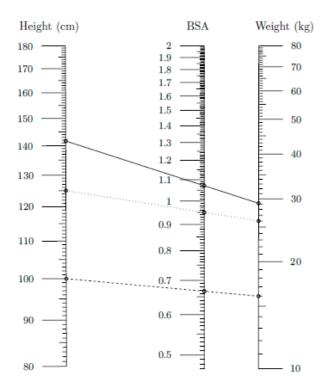
OR

$$30 = \frac{150Y}{Y+12} \Rightarrow 30Y + 360 = 150Y \Rightarrow Y = 3$$

The child is 3 years old.

(f) Another rule for working out a child's dose is based on "body surface area" (BSA). The rule is:

$$\text{child's dose} = \frac{\text{child's BSA in m}^2}{1\cdot73} \times \text{adult dose}$$

BSA is difficult to measure directly, but an estimate can be calculated from a person's height and weight. The chart below allows you to read off the BSA for a given height and weight, by drawing a straight line from the height on the left scale to the weight on the right. For example, the dotted line shows that a person of height 100 cm and weight 16 kg has a BSA of $0\cdot67$ m^2.

The correct adult dose of a certain medicine is 200 mg per day. Use the BSA rule to calculate the correct dose for a child of height 125 cm and weight 26 kg.

BSA = 0·95 (reading from chart)

$$C = \frac{BSA}{1\cdot73} \times A$$

$$= \frac{0\cdot95}{1\cdot73} \times 200$$

$$\approx 109\cdot83 \text{ mg} \approx 110 \text{ mg}.$$

(g) The following apply in the case of a certain medicine and a certain child:

- the child is nine years old
- Clark's rule and Young's rule both give a dose of 90 mg per day
- the BSA rule gives a dose of 130 mg per day.

Find the weight and height of this child.

Young's rule: $C = \dfrac{YA}{Y+12} \Rightarrow \dfrac{9A}{9+12} = 90 \Rightarrow A = \dfrac{21}{9} \times 90 = 210\,\text{mg}$

Clark's rule: $C = \dfrac{W}{68} \times A \Rightarrow \dfrac{W}{68} \times 210 = 90 \Rightarrow W = \dfrac{90 \times 68}{210} = 29\dfrac{1}{7}\,\text{kg}$

Weight of child is $29\dfrac{1}{7}\,\text{kg}$

$C = \dfrac{BSA}{1\cdot73} \times A \Rightarrow \dfrac{BSA}{1\cdot73} \times 210 = 130 \Rightarrow BSA = \dfrac{130 \times 1\cdot73}{210} \approx 1\cdot07\,\text{m}^2$

$\therefore$ Height: 142 cm (using chart)