Maths
Leaving Certificate
Ordinary Level

Past Exam Questions
Marking Scheme on
Calculus - Differentiation

Q5 2013 paper One Section A

Question 5

(a) Let $y = 2x^3 - 3x^2 - 1$. Find $\frac{dy}{dx}$.

$$y = 2x^3 - 3x^2 - 1 \Rightarrow \frac{dy}{dx} = 6x^2 - 6x.$$

(b) **(i)** Differentiate $(2x^2 + 3x + 1)(x^3 - x + 2)$ with respect to x.

$$y = \left(2x^2 + 3x + 1\right)\left(x^3 - x + 2\right)$$

$$\text{Let } u = 2x^2 + 3x + 1 \Rightarrow \frac{du}{dx} = 4x + 3$$

$$\text{Let } v = x^3 - x + 2 \Rightarrow \frac{dv}{dx} = 3x^2 - 1$$

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= \left(2x^2 + 3x + 1\right)\left(3x^2 - 1\right) + \left(x^3 - x + 2\right)\left(4x + 3\right)$$

$$= 6x^4 - 2x^2 + 9x^3 - 3x + 3x^2 - 1 + 4x^4 + 3x^3 - 4x^2 - 3x + 8x + 6$$

$$= 10x^4 + 12x^3 - 3x^2 + 2x + 5$$

(ii) Let $y = \frac{3x}{2x+5}$, where $2x + 5 \neq 0$. Find the value of $\frac{dy}{dx}$ at $x = 0$.

$$y = \frac{3x}{2x+5}$$

$$\text{Let } u = 3x \Rightarrow \frac{du}{dx} = 3$$

$$\text{Let } v = 2x + 5 \Rightarrow \frac{dv}{dx} = 2$$

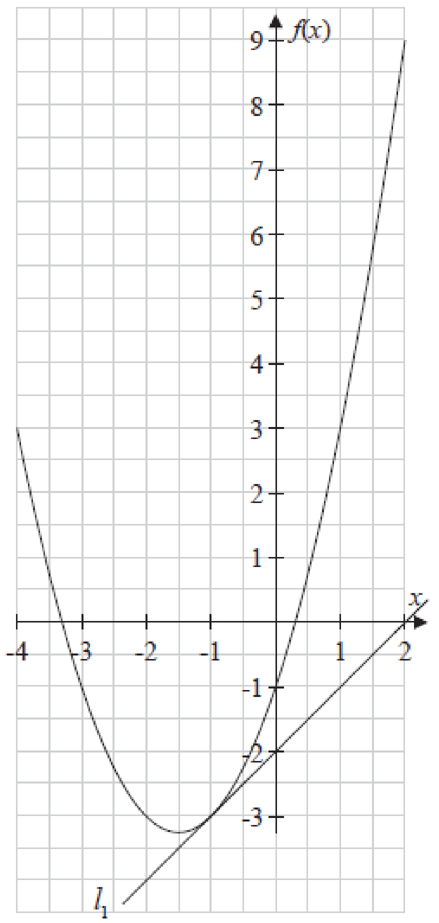
$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(2x+5)(3) - (3x)(2)}{(2x+5)^2}$$

$$= \frac{(0+5)(3) - (0)(2)}{(0+5)^2} = \frac{15 - 0}{25} = \frac{3}{5} \quad \text{at } x = 0$$

Q6 2013 Section A

Question 6 **(25 marks)**

The diagram opposite shows graphs of the quadratic function $f(x) = x^2 + 3x - 1$, $x \in \mathbb{R}$ and the line l_1.
The line l_1 passes through the point (2, 0) and is a tangent to the curve at the point $(-1, -3)$.



(a) Find the slope of l_1, using a slope formula.

$$\text{Slope } = \frac{-3-0}{-1-2} = \frac{-3}{-3} = 1$$

(b) (i) Find $f'(x)$, the derivative of $f(x)$.

$$f(x) = x^2 + 3x - 1$$
$$f'(x) = 2x + 3$$

(ii) Verify your answer to (a) above by finding the value of $f'(x)$ at $x = -1$.

$$f'(x) = 2x + 3 \quad \Rightarrow \quad f'(-1) = 2(-1) + 3 = 1 \ \text{ at } \ x = -1$$

(c) The line l_2 is perpendicular to l_1 and is also a tangent to the curve $f(x)$. Find the co-ordinates of the point at which l_2 touches the curve.

$l_1 \perp l_2$: Slope of $l_1 = 1 \quad \Rightarrow \quad$ slope of $l_2 = -1$

$f'(x) = 2x + 3 = -1 \quad \Rightarrow \quad 2x = -4 \quad \Rightarrow \quad x = -2$

$f(x) = x^2 + 3x - 1 \quad \Rightarrow \quad f(-2) = (-2)^2 + 3(-2) - 1 = -3$

Point of contact $(-2, -3)$