Maths
Leaving Certificate
Ordinary Level

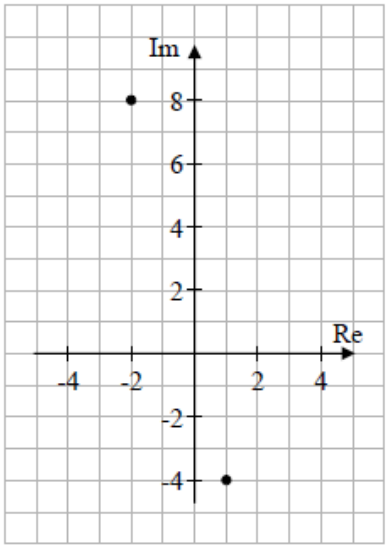
Past Exam Questions
Marking Scheme on
Complex Numbers

Q3 2012 Ordinary Level Section A

Question 3

Question 3 **(25 marks)**

The complex number $z = 1 - 4i$, where $i^2 = -1$.



(a) Plot z and $-2z$ on the Argand diagram.

$$-2z = -2(1-4i) = -2+8i$$

(b) Show that $2|z| = |-2z|$.

$$2|z| = 2|1-4i| = 2\sqrt{1^2+(-4)^2} = 2\sqrt{17}$$

$$|-2z| = |-2+8i| = \sqrt{(-2)^2+8^2} = \sqrt{68} = 2\sqrt{17}$$

$$\therefore 2|z| = |-2z|$$

(c) What does part **(b)** tell you about the points you plotted in part **(a)**?

$-2z$ is twice as far from the origin as z is.

(d) Let k be a real number such that $|z+k| = 5$. Find the two possible values of k.

$$|z+k| = 5 \Rightarrow |1-4i+k| = 5$$
$$\Rightarrow |(1+k)-4i| = 5 \Rightarrow \sqrt{(1+k)^2+(-4)^2} = 5$$
$$\Rightarrow (1+k)^2 + 16 = 25$$
$$\Rightarrow (1+k)^2 = 9 \Rightarrow 1+k = \pm 3 \Rightarrow k = 2 \text{ or } k = -4$$

OR

$$(1+k)^2 + 16 = 25 \Rightarrow 1+2k+k^2+16-25 = 0 \Rightarrow k^2+2k-8 = 0$$
$$\Rightarrow (k-2)(k+4) = 0 \Rightarrow k = 2 \text{ or } k = -4$$