# Maths
# Leaving Certificate
# Ordinary Level

# Past Exam Questions
# Marking Scheme on
# Graphing Functions

## Q2 2013 Sample paper 1

**Question 2** **(25 marks)**

The diagram shows the graph of the function $f(x) = 6x - x^2$ in the domain $0 \le x \le 6$, $x \in \mathbb{R}$.

**(a)** Find $f(0)$, $f(1)$, $f(2)$, $f(3)$, $f(4)$, $f(5)$ and $f(6)$.
Hence, complete the table below.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | **0** | **5** | **8** | **9** | **8** | **5** | **0** |

$f(x) = 6x - x^2$

$f(0) = 6(0) - 0^2 = 0$

$f(1) = 6(1) - 1^2 = 5$

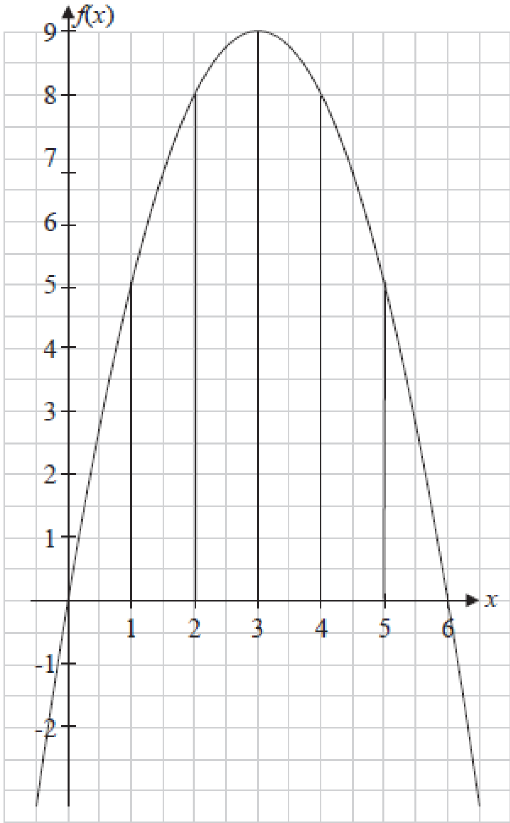
$f(2) = 6(2) - 2^2 = 8$

$f(3) = 6(3) - 3^2 = 9$

$f(4) = 6(4) - 4^2 = 8$

$f(5) = 6(5) - 5^2 = 5$

$f(6) = 6(6) - 6^2 = 0$




**(b)** Use the trapezoidal rule to estimate the area of the region enclosed between the curve and the $x$-axis in the given domain.

$$A \approx \frac{h}{2}[y_1 + y_n + 2(y_2 + y_3 + y_4 + \cdots + y_{n-1})]$$

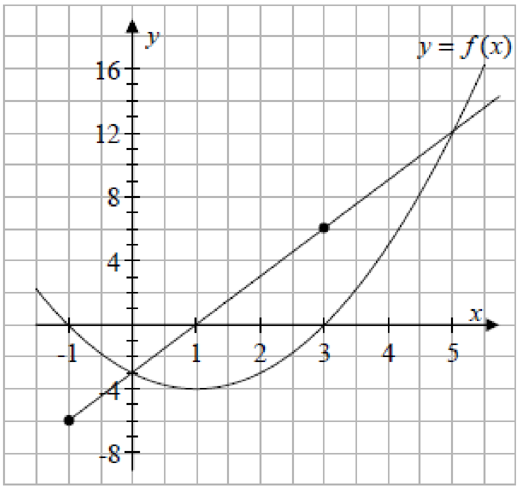
$$= \frac{1}{2}[0 + 0 + 2(5 + 8 + 9 + 8 + 5)]$$

$$= 35$$

## Q5 2012 Paper 1

**Question 5** **(25 marks)**

The diagram shows the graph of a function $f$.




**(a)** The graph of another function $g$ is a straight line.

$g(-1) = -6$ and $g(3) = 6$.

Draw the graph of $g$ on the diagram.

**(b)** Use the graphs to find the two values of $x$ for which $g(x) = f(x)$.

$x = 0$ or $x = 5$

**(c)** The functions $g$ and $f$ are defined for $x \in \mathbb{R}$ by:

$g : x \mapsto ax + b$

$f : x \mapsto x^2 + px + q$

where $a$, $b$, $p$, and $q$ are constants.
The graph of $f$ crosses the $x$-axis at $-1$ and 3, as shown.
By finding the values of $a$, $b$, $p$, and $q$, use algebra to solve $g(x) = f(x)$.

$g(x) = ax + b$

$g(-1): -a + b = -6$

$g(3): \quad 3a + b = 6$

$\Rightarrow \; -4a = -12 \;\Rightarrow\; a = 3 \qquad \therefore -3 + b = -6$

$b = -3$

$\Rightarrow g(x) = 3x - 3$

$f: \quad x = -1, x = 3 \;\Rightarrow\; f(x) = (x+1)(x-3) = x^2 - 2x - 3$

**OR**

$f(x) = x^2 + px + q$

$f(-1): \quad 1 - p + q = 0$

$f(3): \quad 9 + 3p + q = 0$

$\Rightarrow \; -4p = 8 \;\Rightarrow\; p = -2 \;\therefore\; q = -3$

$\Rightarrow \; f(x) = x^2 - 2x - 3$

$g(x) = f(x) \Rightarrow 3x - 3 = x^2 - 2x - 3 \Rightarrow x^2 - 5x = 0$

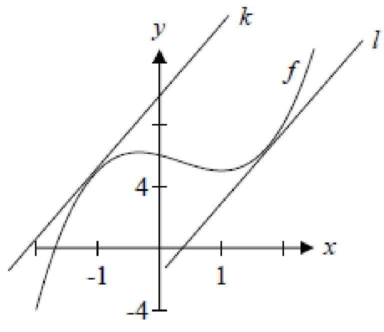
$\Rightarrow x(x-5) = 0 \Rightarrow x = 0$ or $x = 5$

## Q6 2012 Paper 1

**Question 6** **(25 marks)**

The diagram shows the graph of the cubic function *f*, defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 - x^2 - x + 6 .$$




**(a)** Find the co-ordinates of the point at which *f* cuts the *y*-axis.

$f(x) = x^3 - x^2 - x + 6$
$f(0) = 0 - 0 - 0 + 6 = 6 \quad \Rightarrow \quad (0, 6)$

**(b)** *f* has a minimum turning point at (1, 5).
Find the co-ordinates of the maximum turning point.

$f(x) = x^3 - x^2 - x + 6$

$f'(x) = 3x^2 - 2x - 1 = 0$
$\Rightarrow \quad (3x+1)(x-1) = 0$
$\Rightarrow \quad x = -\frac{1}{3} \text{ or } x = 1$

$f(-\frac{1}{3}) = (-\frac{1}{3})^3 - (-\frac{1}{3})^2 - (-\frac{1}{3}) + 6$
$\quad = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 6 = 6\frac{5}{27}$

Maximum turning point $\left(-\frac{1}{3}, 6\frac{5}{27}\right)$.

**(c)** The lines *k* and *l* are tangents to the curve $y = f(x)$ and *l* is parallel to *k*. The equation of *k* is $4x - y + 9 = 0$. Find the *x* co-ordinate of the point at which *l* is a tangent to the curve.

$k : 4x - y + 9 = 0 \quad \Rightarrow \quad y = 4x + 9$
Therefore, the slope of $l = 4$.

$f'(x) = 3x^2 - 2x - 1 = 4$
$\Rightarrow \quad 3x^2 - 2x - 5 = 0$
$\Rightarrow \quad (3x-5)(x+1) = 0$
$\Rightarrow \quad x = \frac{5}{3} \text{ or } x = -1$

$x = \frac{5}{3}$