



Maths
Leaving Certificate
Ordinary Level

Past Exam Questions
Marking Scheme on
Sequences and Series

Q8 2013 Paper 1 Section B

Question 8

(60 marks)

Two brothers, Eoin and Peter, began work in 2005 on starting salaries of €20 000 and €17 000 per annum, respectively. Eoin's salary increased by €500 per annum and Peter's salary increased by €1250 per annum. This salary pattern will continue.

(a) Complete the table, showing the annual salary of each brother for the years 2005 to 2010.

Year	1	2	3	4	5	6
Eoin's salary (€)	20 000	20 500	21 000	21 500	22 000	22 500
Peter's salary (€)	17 000	18 250	19 500	20 750	22 000	23 250

(b) In what year will both brothers earn the same amount?

Answer: 2009

(c) Eoin claims that their salaries over the years can be represented by an arithmetic sequence.

(i) Explain what an arithmetic sequence is.

A sequence in which the difference between any two successive terms is a constant.

(ii) Do you agree with Eoin? Explain your answer.

Answer: Yes.

A constant amount is added to his salary in any year to give his salary for the following year.

(d) Find, in terms of n , a formula that gives Eoin's salary in the n^{th} year of the pattern.

$$T_n = 20\,000 + (n - 1)500 = 19\,500 + 500n$$

(e) Using your formula, or otherwise, find Eoin's salary in 2015.

$$\begin{aligned} 2015: \quad n &= 11 \\ T_{11} &= 19\,500 + 500(11) = €25\,000 \end{aligned}$$

- (f) Find, in terms of n , a formula that gives the total amount earned by Peter from the first to the n^{th} year of the pattern.

The first term: 17 000
 The common difference: 1250

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(34\,000 + (n-1)1250) = 625n^2 + 16375n$$

- (g) Using your formula, or otherwise, find the total amount earned by Peter from the start of 2005 up to the end 2015.

$$S_n = 625n^2 + 16375n = 625(11)^2 + 16375(11) = 75\,625 + 180\,125 = \text{€}255\,750$$

or

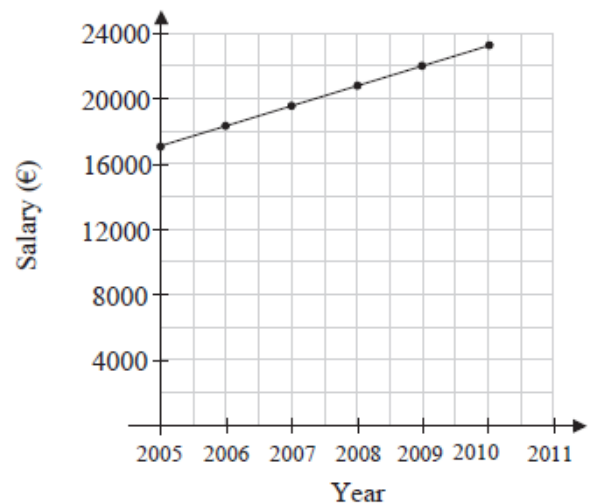
$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{11} = \frac{11}{2}(34\,000 + (11-1)1250) = \frac{11}{2}(34\,000 + 12\,500) = \text{€}255\,750$$

- (h) Give one reason why the graph below is not an accurate way to represent Peter's salary over the period 2005 to 2011.

The graph shows Peter's salary increasing constantly throughout the year which is not true.

Peter's salary increases in steps at the end of each year and the graph does not reflect this.

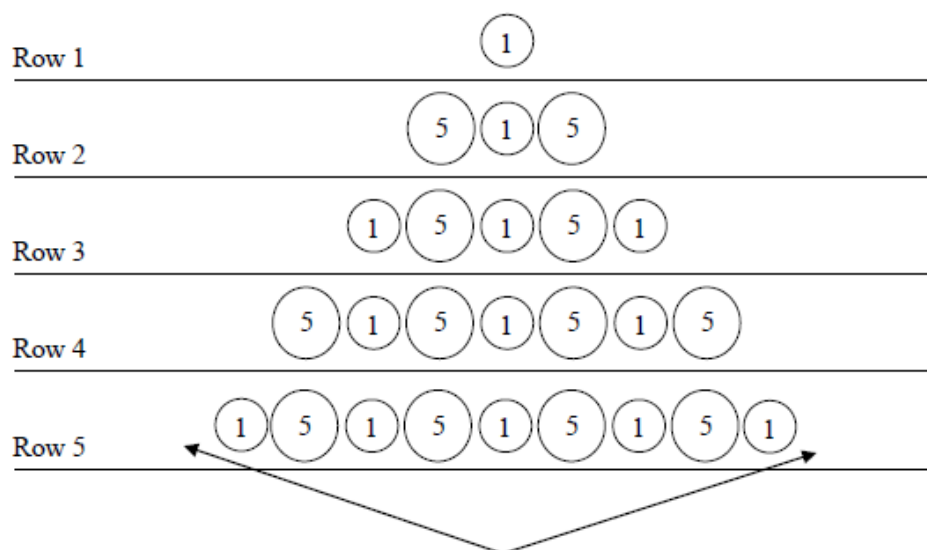


Q8 2012

Question 8

(50 marks)

Lucy is arranging 1 cent and 5 cent coins in rows. The pattern of coins in each row is as shown below.



(a) Draw the next row of coins above, continuing the same pattern.

(b) The table below gives the number of coins and the total value of the coins in each row. Complete the table for rows 4 to 7.

<i>Row number n</i>	<i>Number of 1 cent coins</i>	<i>Number of 5 cent coins</i>	<i>Total number of coins in the row</i>	<i>Total value of the coins in the row</i>
1	1	0	1	1
2	1	2	3	11
3	3	2	5	13
4	3	4	7	23
5	5	4	9	25
6	5	6	11	35
7	7	6	13	37

(c) Complete the following sentences to state, in terms of n , the number of 1 cent and 5 cent coins in row n .

(i) If n is odd, row n has n 1 cent coins and $(n-1)$ 5 cent coins.

(ii) If n is even, row n has $(n-1)$ 1 cent coins and n 5 cent coins.

(d) Find the total number of coins in the 40th row.

$n = 40$ which is an even number

\therefore Number of coins: $n - 1 + n = 2n - 1 = 80 - 1 = 79$ coins

OR

$$T_n = a + (n-1)d$$

$$T_{40} = 1 + (40-1)2$$

$$T_{40} = 79$$

(e) Find the total value of the coins in the 40th row.

$$\begin{aligned} \text{Total Value:} &= (n-1)(1) + n(5) \\ &= (40-1)(1) + 40(5) \\ &= 39 + 200 \\ &= 239 \end{aligned}$$

OR

$$\begin{aligned} \text{Even Rows: } &11, 23, 35, \dots \\ &a = 11, d = 12, n = 20 \\ &T_{20} = 11 + 19(12) \\ &T_{20} = 239 \end{aligned}$$

Total Value: 239 cent

- (f) Which row has coins with a total value of 337 cent?

$$\text{If } n \text{ odd: } (n)(1) + (n-1)(5) = 337$$

$$\Rightarrow n + 5n - 5 = 337$$

$$\Rightarrow 6n = 342$$

$$\Rightarrow n = 57$$

$$\text{If } n \text{ even: } (n-1)(1) + n(5) = 337$$

$$\Rightarrow 6n - 1 = 337$$

$$\Rightarrow 6n = 338$$

$$\Rightarrow n = 56\frac{1}{3} \notin \mathbb{N}.$$

\therefore Row 57 has a total value of 337 cent.

- (g) Find the total value of the coins in the first 40 rows.

$$\begin{aligned} S &= 1 + 11 + 13 + 23 + 25 + 35 + \dots + 239 \\ &= (1 + 13 + 25 + \dots) + (11 + 23 + 35 + \dots) \\ &= \frac{20}{2}(2 + (20-1)12) + \frac{20}{2}(22 + (20-1)12) \\ &= 10(230) + 10(250) = 4800 \text{ cent.} \end{aligned}$$

OR

Number of 1 cent coins:

$$\begin{aligned} S &= 1 + 1 + 3 + 3 + 5 + 5 + \dots + 39 + 39 \\ &= 2(1 + 3 + 5 + \dots) = 2\left[\frac{20}{2}(2 + (20-1)2)\right] = 800 \text{ coins} \end{aligned}$$

Number of 5 cent coins:

$$\begin{aligned} S &= 0 + 2 + 2 + 4 + 4 + \dots + 38 + 40 \\ &= (0 + 2 + 4 + \dots) + (2 + 4 + 6 + \dots) \\ &= \frac{20}{2}(0 + (20-1)2) + \frac{20}{2}(4 + (20-1)2) = 380 + 420 = 800 \text{ coins} \end{aligned}$$

$$\text{Value: } 800(1) + 800(5) = 4800 \text{ cent}$$