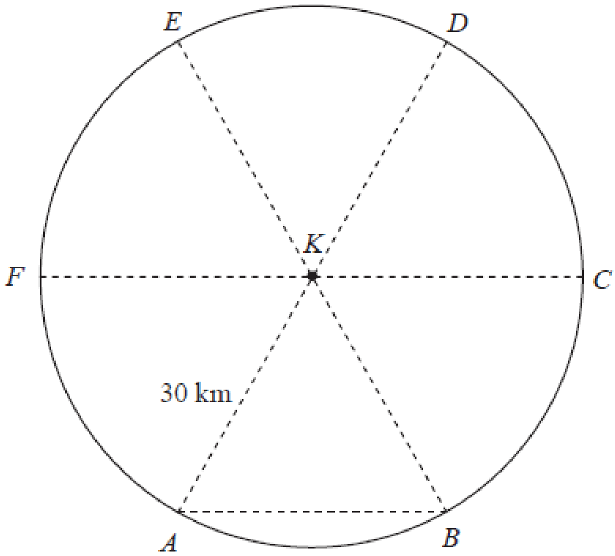
Maths
Leaving Certificate
Ordinary Level

Past Exam Questions
Marking Scheme on
Trigonometry

Q8 2013 Project Maths Paper Two Ordinary Level Section B

Question 8 **(75 marks)**

A search is begun for a buoy that has become detached from its mooring at sea. The area to be searched is a circle of radius 30 km from the last known position, K, of the buoy. The search area is divided into six equal sectors as indicated by the letters A, B, C, D, E and F.



(a) Fishing boats search the triangular area KAB.

(i) Find $|\angle BKA|$.

$|\angle BKA| = 360° \div 6 = 60°$

(ii) Find the area of the triangle KAB.

Area $= \frac{1}{2}ab\sin C = \frac{1}{2} \times 30 \times 30 \times \sin 60° = 389 \cdot 7 \text{ km}^2$

(iii) Write the area of the triangle *KAB* as a percentage of the area of the sector *KAB*.

Area of sector: $\frac{1}{6}\pi r^2 = \frac{1}{6}\pi(30)^2 = 471\cdot 24 \text{ km}^2$

$$\frac{389\cdot 7}{471\cdot 24}\times 100 = 82\cdot 7\%$$

(iv) Use the cosine rule to find the length of $[AB]$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$|AB|^2 = 30^2 + 30^2 - 2(30)(30)\cos 60° = 900 \quad \Rightarrow \quad |AB| = 30 \text{ km}$$

(v) What does your answer to **(iv)** above show about the triangle *KAB*?

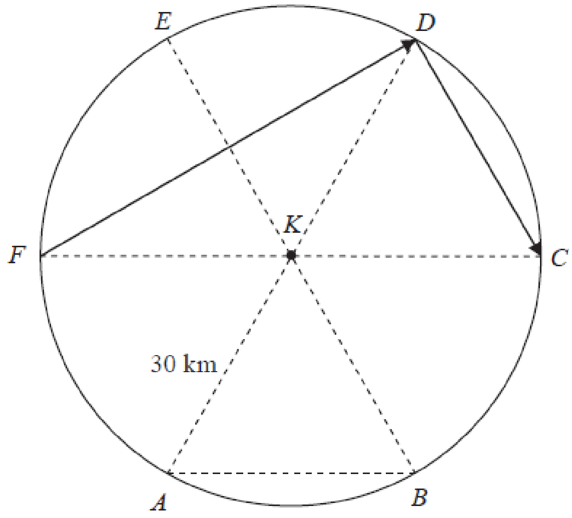
The triangle *KAB* is an equilateral triangle.

(b) A helicopter took part in the search.

(i) The helicopter flew from the point F around the perimeter of the search area. What distance did the helicopter fly, correct to the nearest km?

$$2\pi r = 2\pi \times 30 = 188 \cdot 49 \approx 188 \text{ km}$$

(ii) The helicopter then flew in a straight line from F to D and from D on to C, also in a straight line. Draw the path of the helicopter on the diagram.



(iii) A theorem on your course can be used to find $|\angle FDC|$.
Write down $|\angle FDC|$ and state the theorem.

$$|\angle FDC| = 90^\circ$$

The angle in a semicircle is a right-angle.

(iv) The helicopter flew at a speed of 80 km/h. How long did it take to fly from F to D and on to C?

$$\cos 30^\circ = \frac{|FD|}{60} \quad \Rightarrow \quad |FD| = 60\cos 30^\circ = 51 \cdot 96 \text{ km}$$

$$|FD| + |DC| = 51 \cdot 96 + 30 = 81 \cdot 96 \text{ km}$$

Time: $\dfrac{81 \cdot 96}{80} = 1 \cdot 0245$ hours

(c) A lifeboat taking part in the search sailed, in a straight line, from the point K until it reached a point X, the midpoint of $[ED]$.

(i) Calculate $|KX|$.

$$\Delta KXE: \quad \cos 30^\circ = \frac{|KX|}{30} \quad \Rightarrow \quad |KX| = 30\cos 30^\circ = 25\cdot 98 \text{ km}$$

Or

$$|KX|^2 + |XE|^2 = |EK|^2$$
$$\Rightarrow \quad |KX|^2 = |EK|^2 - |XE|^2 = 30^2 - 15^2 = 675 \quad \Rightarrow \quad |KX| = 25\cdot 98 \text{ km}$$

(ii) The buoy was located at the point where the path KX, of the lifeboat, crossed the path FD of the helicopter. How far was the buoy from X?

$$KX \cap FD = \{P\}$$

$$\Delta DXP: \quad \tan 30^\circ = \frac{|XP|}{15} \quad \Rightarrow \quad |XP| = 15\tan 30^\circ = 8\cdot 66 \text{ km}$$

Or

$$\Delta FKP: \quad \tan 30^\circ = \frac{|KP|}{30} \quad \Rightarrow \quad |KP| = 30\tan 30^\circ = 17\cdot 32 \text{ km}$$

$$|XP| = 25\cdot 98 - 17\cdot 32 = 8\cdot 66 \text{ km}$$

Q8 2012 Project Maths Paper Two Ordinary Level Section B

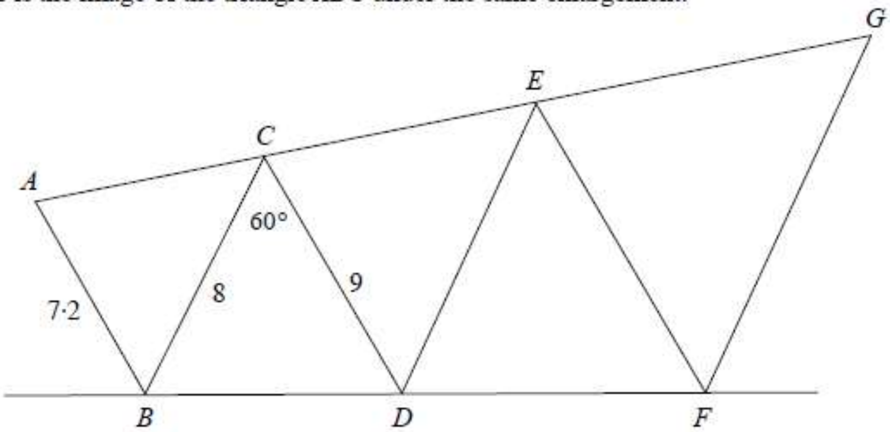
Question 8 **(75 marks)**

(a) The planned supports for the roof of a building form scalene triangles of different sizes.

(i) Explain what is meant by a **scalene triangle**.

A triangle in which the three sides have different lengths

The triangle *EFG* is the image of the triangle *CDE* under an enlargement and the triangle *CDE* is the image of the triangle *ABC* under the same enlargement.



The proposed dimensions for the structure are $|AB| = 7\cdot2$ m, $|BC| = 8$ m, $|CD| = 9$ m and $|\angle DCB| = 60°$.

(ii) Find the length of [*FG*].

Scale factor $= \frac{9}{7\cdot2} = 1\cdot25$

$|DE| = 1\cdot25|BC| = 1\cdot25(8) = 10 \Rightarrow |FG| = 1\cdot25|DE| = 1\cdot25(10) = 12\cdot5$ m

(iii) Find the length of [*BD*], correct to three decimal places.

$|BD|^2 = 8^2 + 9^2 - (2)(8)(9)\cos 60° = 64 + 81 - 72 = 73$

$\Rightarrow |BD| = \sqrt{73} = 8\cdot544$ m

(iv) The centre of the enlargement is O. Find the distance from O to the point B.

$$\frac{|OD|}{|OB|} = \frac{x + 8{\cdot}544}{x} = 1{\cdot}25$$

$\Rightarrow \quad x + 8{\cdot}544 = 1{\cdot}25x$

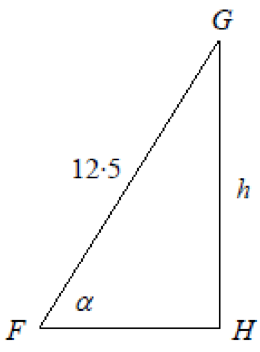
$\Rightarrow \quad 0{\cdot}25x = 8{\cdot}544$

$\Rightarrow \quad x = 34{\cdot}176$ m

(v) A condition of the planning is that the height of the point G above the horizontal line BF cannot exceed 11·6 m.

Does the plan meet this condition? Justify your answer by calculation.

$|\angle GFH| = \alpha = |\angle CBD|$



In triangle CBD:

$$\frac{\sin\alpha}{9} = \frac{\sin 60^\circ}{8{\cdot}544} \Rightarrow \sin\alpha = \frac{9\sin 60^\circ}{8{\cdot}544}$$

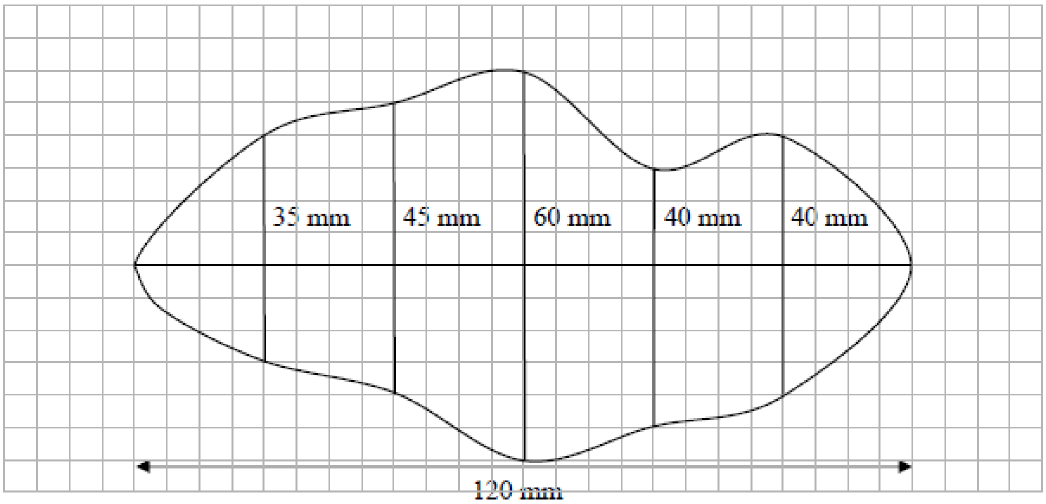
In triangle GFH:

$$\sin\alpha = \frac{h}{12{\cdot}5} = \frac{9\sin 60^\circ}{8{\cdot}544}$$

$$\Rightarrow h = \frac{12{\cdot}5 \times 9\sin 60^\circ}{8{\cdot}544} = 11{\cdot}4 < 11{\cdot}6$$

Yes, the plan meets the condition.

(b) In order to estimate the area of the irregular shape shown below, a horizontal line was drawn across the widest part of the shape and five offsets (perpendicular lines) were drawn at equal intervals along this line.



(i) Find the lengths of the horizontal line and the offsets, taking each grid unit as 5 mm, and record the lengths on the diagram.

(ii) Use the trapezoidal rule to estimate the area of the shape.

$$
\begin{aligned}
A &= \tfrac{20}{2}\left(0 + 0 + 2(35 + 45 + 60 + 40 + 40)\right) \\
&= \tfrac{20}{2}(440) \\
&= 4400 \text{ mm}^2
\end{aligned}
$$