



Integration Q8
Maths Past Exam Questions
Higher Level

Paper 1 – Section A – Q6

8. (a) Find $\int (1 + \cos 2x + e^{3x}) dx$.

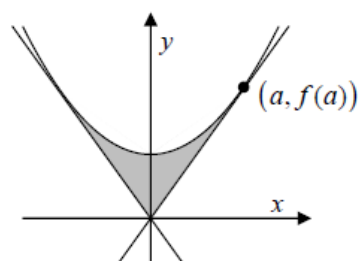
(b) (i) Evaluate $\int_1^3 \frac{12}{3x-2} dx$.

(ii) Evaluate $\int_0^{\frac{\pi}{8}} \sin^2 2x dx$.

(c) The function f is given by $f(x) = x^2 + k$, where k is a positive constant.

(i) The tangent to the curve $y = f(x)$ at the point $(a, f(a))$ passes through the origin, where $a > 0$. Express a in terms of k .

(ii) The tangent at $(-a, f(-a))$ also passes through the origin. Find, in terms of k , the area of the region enclosed by these two tangents and the curve.



2011

8. (a) Find $\int (x^3 + \sqrt{x}) dx$.

(b) (i) Evaluate $\int_0^2 \frac{x+1}{x^2+2x+2} dx$.

(ii) Evaluate $\int_0^2 \frac{x^2+2x+2}{x+1} dx$.

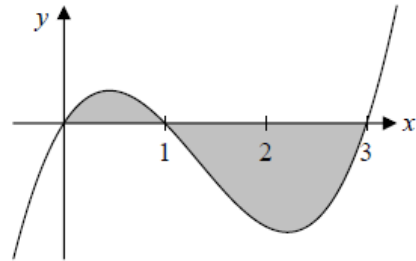
(c) Use integration methods to establish the formula $A = \pi r^2$ for the area of a disc of radius r .

2010

8. (a) Find $\int (\sin 2x + e^{4x}) dx$.

- (b) The curve $y = 12x^3 - 48x^2 + 36x$ crosses the x -axis at $x = 0$, $x = 1$ and $x = 3$, as shown.

Calculate the total area of the shaded regions enclosed by the curve and the x -axis.



- (c) (i) Find, in terms of a and b ,

$$I = \int_a^b \frac{\cos x}{1 + \sin x} dx$$

- (ii) Find in terms of a and b ,

$$J = \int_a^b \frac{\sin x}{1 + \cos x} dx.$$

- (iii) Show that if $a + b = \frac{\pi}{2}$, then $I = J$.

2009

8. (a) Find $\int \left(6x + 3 + \frac{1}{x^2} \right) dx$.

(b) Evaluate (i) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin 3x \sin x \, dx$ (ii) $\int_{\ln 3}^{\ln 8} e^x \sqrt{1 + e^x} \, dx$.

(c) Use integration methods to establish the standard formula for the volume of a cone.

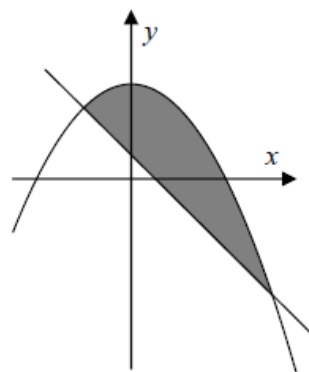
2008

8. (a) Find $\int (2x + \cos 3x) dx$.

(b) Evaluate (i) $\int_0^1 3x^2 e^{x^3} dx$ (ii) $\int_2^4 \frac{2x^3}{x^2 - 1} dx$.

(c) The diagram shows the curve $y = 4 - x^2$ and the line $2x + y - 1 = 0$.

Calculate the area of the shaded region enclosed by the curve and the line.



2007

8. (a) Find (i) $\int x^3 dx$ (ii) $\int \frac{1}{x^3} dx$.

(b) (i) Evaluate $\int_0^4 x\sqrt{x^2+9} dx$.

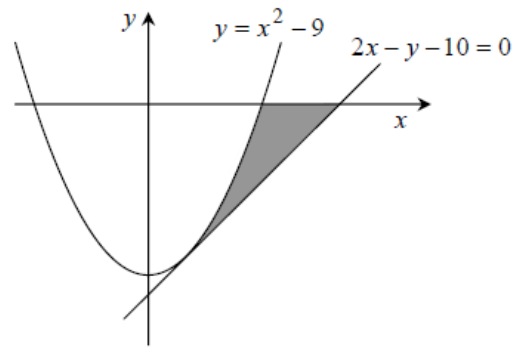
(ii) f is a function such that $f'(x) = 6 - \sin x$ and $f\left(\frac{\pi}{3}\right) = 2\pi$.

Find $f(x)$.

(c) The line $2x - y - 10 = 0$ is a tangent to the curve $y = x^2 - 9$, as shown.

The shaded region is bounded by the line, the curve and the x -axis.

Calculate the area of this region.



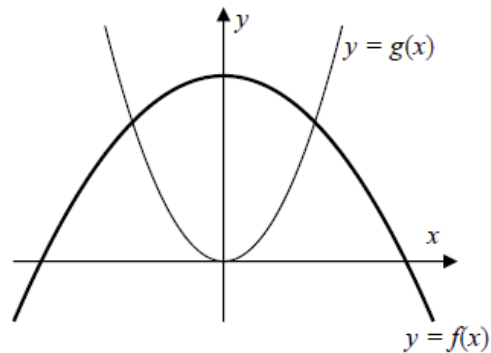
2006

8. (a) Find (i) $\int \sqrt{x} dx$ (ii) $\int e^{-2x} dx$.

(b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$ (ii) $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta d\theta$.

(c) The diagram shows the graphs of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = 12 - 3x^2$ and $g(x) = 9x^2$.

- (i) Calculate the area of the region enclosed by the curve $y = f(x)$ and the x -axis.
- (ii) Show that the region enclosed by the curves $y = f(x)$ and $y = g(x)$ has half that area.



2005

8. (a) Find (i) $\int (2 + x^3) dx$ (ii) $\int e^{3x} dx$.

(b) (i) Evaluate $\int_1^4 \frac{2x+1}{x^2+x+1} dx$.

(ii) Evaluate $\int_0^{\frac{\pi}{8}} \sin^2 2\theta d\theta$.

(c) (i) Evaluate $\int_1^2 \frac{1}{\sqrt{3+2x-x^2}} dx$.

(ii) Use integration methods to derive a formula for the volume of a cone.