Algebra

Maths Past Exam Questions

Marking Schemes

Higher Level

2013

Paper 1 – Project Maths – Section A – Q2

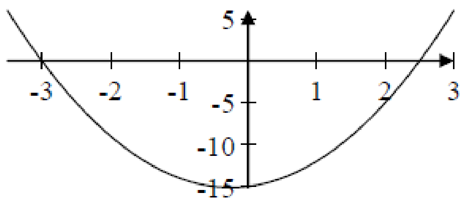
Question 2 **(25 marks)**

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

$2x^2 + x - 15 = 0$

$\Rightarrow (2x-5)(x+3) = 0 \Rightarrow x = 2\tfrac{1}{2}$ or $x = -3$

$2x^2 + x - 15 \geq 0$ for $\{x \mid x \leq -3\} \cup \{x \mid x \geq 2\tfrac{1}{2}\}$



OR

$f(x) = 2x^2 + x - 15 = (2x-5)(x+3)$

$(2x-5)(x+3) = 0$

$\Rightarrow x = \dfrac{5}{2}$ or $x = -3$

(i): $x \geq -3$ and $x \geq \dfrac{5}{2} \Rightarrow x \geq \dfrac{5}{2}$

(ii): $x \leq -3$ and $x \leq \dfrac{5}{2} \Rightarrow x \leq -3$

Solution Set: $\{x \mid x \leq -3\} \cup \{x \mid x \geq \tfrac{5}{2}\}$

(b) Solve the simultaneous equations,

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

$$\begin{aligned} x + y + z &= 16 \\ \tfrac{5}{2}x + y + 10z &= 40 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 2x + 2y + 2z &= 32 \\ 5x + 2y + 20z &= 80 \\ \hline 3x \quad + 18z &= 48 \end{aligned}$$

$$\begin{aligned} x + y + z &= 16 \\ 4x + y + 8z &= 42 \\ \hline 3x \quad + 7z &= 26 \end{aligned}$$

$$\begin{aligned} 3x + 18z &= 48 \\ 3x \ + 7z &= 26 \\ \hline 11z &= 22 \quad \Rightarrow \quad z = 2 \end{aligned}$$

$3x + 7z = 26 \Rightarrow 3x + 7(2) = 26 \Rightarrow 3x = 12 \Rightarrow x = 4$

$x + y + z = 16 \Rightarrow 4 + y + 2 = 16 \Rightarrow y = 10$

Paper 1 – Project Maths – Section B – Q7

Question 7 **(50 marks)**

A stadium can hold 25 000 people. People attending a regular event at the stadium must purchase a ticket in advance. When the ticket price is €20, the expected attendance at an event is 12 000 people. The results of a survey carried out by the owners suggest that for every €1 reduction, from €20, in the ticket price, the expected attendance would increase by 1000 people.

(a) If the ticket price was €18, how many people would be expected to attend?

$$12000+(20-18)1000=14000$$

(b) Let x be the ticket price, where $x \le 20$. Write down, in terms of x, the expected attendance at such an event.

$$12000+(20-x)1000=32000-1000x$$

(c) Write down a function f that gives the expected income from the sale of tickets for such an event.

$$f(x)=(32000-1000x)x$$

(d) Find the price at which tickets should be sold to give the maximum expected income.

$$f(x)=(32000-1000x)x$$

$$f'(x)=32000-2000x=0 \Rightarrow x=€16$$

(e) Find this maximum expected income.

$$f(x)=(32000-1000x)x$$

$$f(16)=(32000-16000)16=€256\,000$$

(f) Suppose that tickets are instead priced at a value that is expected to give a full attendance at the stadium. Find the difference between the income from the sale of tickets at this price and the maximum income calculated at **(e)** above.

$$32000 - 1000x = 25000 \Rightarrow 1000x = 7000 \Rightarrow x = 7$$

$$f(x) = (32000 - 1000x)x \Rightarrow f(7) = (32000 - 7000)7 = 175\,000$$

Difference: €256 000 − €175 000 = €81 000

(g) The stadium was full for a recent special event. Two types of tickets were sold; a single ticket for €16 and a family ticket (2 adults and 2 children) for a certain amount. The income from this event was €365 000. If 1000 more family tickets had been sold the income from the event would have been reduces by €14 000. How many family tickets were sold?

Single ticket: €16; Family ticket €y
Number of single tickets: p; Number of family tickets: $\frac{25000-p}{4}$

$$16p + \tfrac{25000-p}{4}y = 365000$$

$$16(p-4000) + (\tfrac{25000-p}{4} + 1000)y = 351000 \Rightarrow 16p + \tfrac{29000-p}{4}y = 415000$$

$$\tfrac{29000-p}{4}y - \tfrac{25000-p}{4}y = 50000 \Rightarrow 4000y = 200000 \Rightarrow y = 50$$

$$16p + \tfrac{25000-p}{4}50 = 365000 \Rightarrow 7p = 105000 \Rightarrow p = 15000$$

Number of family tickets: $\frac{25000-p}{4} = \frac{25000-15000}{4} = 2500$

OR

x = number of single tickets
f = number of family tickets
y = cost of family ticket

$$x + 4f = 25000$$
$$16x + fy = 365000$$
$$16(x-4000) + (f+1000)y = 351000$$

$$16x - 64000 + fy + 1000y = 351000$$
$$16x \quad + fy \quad = 365000$$

$$1000y = 50000$$
$$y = 50$$

$$
\begin{array}{r}
x + 4f = 25000 \\
16x + 50f = 365000 \\
\underline{16x + 64f = 400000} \\
14f = 35000 \\
f = 2500
\end{array}
$$

2012

Paper 1 – Project Maths – Section A – Q1

Question 1 **(25 marks)**

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

$$a = -2b - 1$$

$$(-2b-1)^2 + (2b+1)b + b^2 = 3$$
$$7b^2 + 5b - 2 = 0$$
$$(7b-2)(b+1) = 0$$
$$b = \tfrac{2}{7} \quad \text{or } b = -1$$
$$a = \tfrac{-11}{7} \quad \text{or } a = 1$$

Solution: $\{b = \tfrac{2}{7} \text{ and } a = \tfrac{-11}{7}\}$ or $\{b = -1 \text{ and } a = 1\}$.

(b) Find the set of all real values of x for which $\dfrac{2x-5}{x-3} \le \dfrac{5}{2}$.

Multiply across by $2(x-3)^2$, which is non-negative:

$$2(x-3)(2x-5) \le 5(x-3)^2$$
$$4x^2 - 22x + 30 \le 5x^2 - 30x + 45$$
$$0 \le x^2 - 8x + 15$$
$$0 \le (x-5)(x-3)$$

$x \ge 5$ or $x < 3$.

OR

$$\frac{2x-5}{x-3} - \frac{5}{2} \le 0$$
$$\frac{2(2x-5) - 5(x-3)}{2(x-3)} \le 0$$
$$\frac{-x+5}{2(x-3)} \le 0$$

	$x < 3$	$3 < x < 5$	$x > 5$
$-x+5$	+	+	–
$x-3$	–	+	+
$\dfrac{-x+5}{2(x-3)}$	–	+	–

$x \ge 5$ or $x < 3$.