Complex Numbers

Maths Past Exam Questions

Marking Schemes

Higher Level

2013

Paper 1 – Project Maths – Section A – Q1

Question 1 **(25 marks)**

$z = \dfrac{4}{1+\sqrt{3}i}$ is a complex number, where $i^2 = -1$.

(a) Verify that z can be written as $1-\sqrt{3}i$.

$$z = \frac{4}{1+\sqrt{3}i} = \frac{4}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = \frac{4-4\sqrt{3}i}{1+3} = 1-\sqrt{3}i$$

OR

If $z = \dfrac{4}{1+\sqrt{3}i} = 1-\sqrt{3}i$

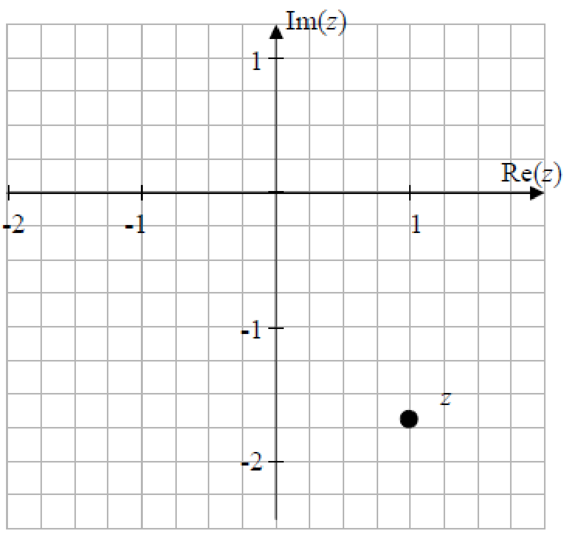
then $4 = (1+\sqrt{3}i)(1-\sqrt{3}i) = (1)^2 + (\sqrt{3})^2 = 4$
$\Rightarrow$ True

(b) Plot z on an Argand diagram and write z in polar form.

$$\tan\alpha = \frac{\sqrt{3}}{1} \Rightarrow \alpha = \frac{\pi}{3} \Rightarrow \theta = \frac{5\pi}{3}$$

$$r = |1-\sqrt{3}i| = \sqrt{1+3} = \sqrt{4} = 2$$

$$z = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$$



(c) Use De Moivre's theorem to show that $z^{10} = -2^9\left(1-\sqrt{3}i\right)$.

$$z^{10} = \left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^{10}$$

$$= 2^{10}\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) = 2^{10}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right)$$

$$= 2^{10}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2^{10}\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -2^9\left(1-\sqrt{3}i\right)$$

2012

Paper 1 – Project Maths – Section A – Q3

Question 3 **(25 marks)**

The complex number z has modulus $5\frac{1}{16}$ and argument $\frac{4\pi}{9}$.

(a) Find, in polar form, the four complex fourth roots of z.
(That is, find the four values of w for which $w^4 = z$.)

$$w^4 = \frac{81}{16}\left(\cos\left(\frac{4\pi}{9} + 2n\pi\right) + i\sin\left(\frac{4\pi}{9} + 2n\pi\right)\right)$$

$$w = \frac{3}{2}\left(\cos\left(\frac{\pi}{9} + \frac{n\pi}{2}\right) + i\sin\left(\frac{\pi}{9} + \frac{n\pi}{2}\right)\right), \quad n = 0, 1, 2, 3.$$

$$w = \frac{3}{2}\left(\cos\left(\frac{\pi}{9}\right) + i\sin\left(\frac{\pi}{9}\right)\right), \quad \frac{3}{2}\left(\cos\left(\frac{11\pi}{18}\right) + i\sin\left(\frac{11\pi}{18}\right)\right),$$

$$\frac{3}{2}\left(\cos\left(\frac{10\pi}{9}\right) + i\sin\left(\frac{10\pi}{9}\right)\right), \quad \frac{3}{2}\left(\cos\left(\frac{29\pi}{18}\right) + i\sin\left(\frac{29\pi}{18}\right)\right)$$

(b) z is marked on the Argand diagram below.
On the same diagram, show the four answers to part **(a)**.