Co-Ordinate Geometry, The Line

Maths Past Exam Questions

Marking Schemes

Higher Level

2013

Paper 2 – Project Maths – Section A – Q3

Question 3 **(25 marks)**

The equations of six lines are given:

Line	Equation
h	$x = 3 - y$
i	$2x - 4y = 3$
k	$y = -\frac{1}{4}(2x - 7)$
l	$4x - 2y - 5 = 0$
m	$x + \sqrt{3}y - 10 = 0$
n	$\sqrt{3}x + y - 10 = 0$

(a) Complete the table below by matching each description given to one or more of the lines.

Description	Line(s)
A line with a slope of 2.	l
A line which intersects the y-axis at $(0, -2\frac{1}{2})$.	l
A line which makes equal intercepts on the axes.	h
A line which makes an angle of $150°$ with the positive sense of the x-axis.	m
Two lines which are perpendicular to each other.	l and k

(b) Find the acute angle between the lines m and n.

Slope of m: $m_1 = -\frac{1}{\sqrt{3}}$

Slope of n: $m_2 = -\sqrt{3}$

$$\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1 m_2} = \pm\frac{-\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}}\left(-\sqrt{3}\right)} = \pm\frac{\frac{-1+3}{\sqrt{3}}}{1+1} = \pm\frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = 30°$$

2012

Paper 2 – Project Maths – Section A – Q1

Question 1 **(25 marks)**

(a) Given the co-ordinates of the vertices of a quadrilateral $ABCD$, describe **three** different ways to determine, using co-ordinate geometry techniques, whether the quadrilateral is a parallelogram.

1. Check whether both pairs of opposite sides have the same slope (slope formula).
2. Check whether both pairs of opposite sides are equal in length (distance formula).
3. Check whether the midpoints of the diagonals coincide (diagonals bisecting each other).
4. Check whether the translation from A to B is the same as the translation from D to C [or equivalent.]
5. Check whether a pair of opposite sides have the same slope and are equal in length (slope and distance formulae).
6. Use slopes and the formula for the angle between two lines to check whether both pairs of opposite angles are equal.
7. Use slopes and the formula for the angle between two lines to check whether $|\angle A| + |\angle B| = 180°$, and $|\angle C| + |\angle B| = 180°$. [or equivalent]

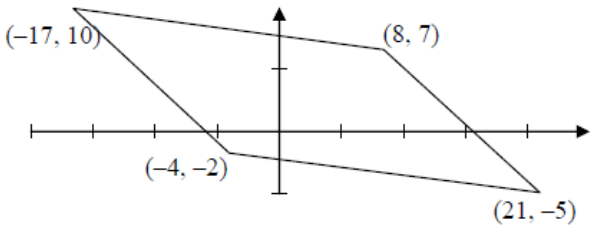
(b) Using **one** of the methods you described, determine whether the quadrilateral with vertices $(-4, -2)$, $(21, -5)$, $(8, 7)$ and $(-17, 10)$ is a parallelogram.

Midpoints of diagonals:

$$\left(\frac{-4+8}{2}, \frac{-2+7}{2}\right) = \left(2, \frac{5}{2}\right)$$

$$\left(\frac{-17+21}{2}, \frac{10-5}{2}\right) = \left(2, \frac{5}{2}\right)$$

Equal $\Rightarrow$ parallelogram.



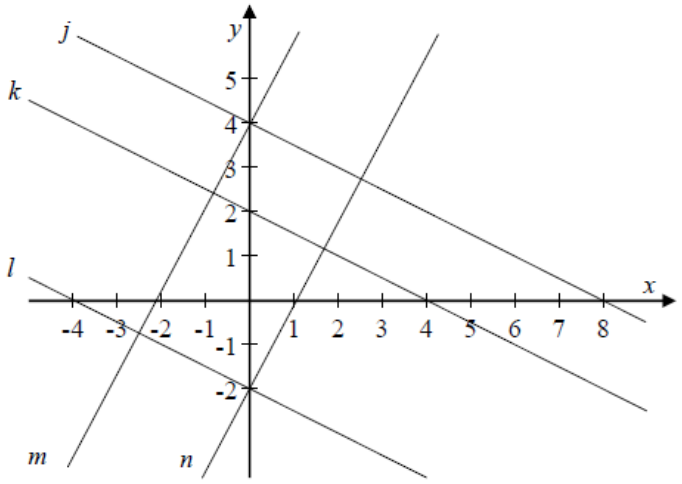
For other methods: slopes are $\frac{-12}{13}$ and $\frac{-3}{25}$; side-lengths are $\sqrt{313}$ and $\sqrt{634}$, translations are $(x, y) \to (x+25, y-3)$ and $(x, y) \to (x+13, y-12)$, or reverse.

2011

Paper 2 – Project Maths – Section A – Q3

Question 3 **(25 marks)**

In the co-ordinate diagram shown, the lines j, k, and l are parallel, and so are the lines m and n.
The equations of four of the five lines are given in the table below.



Equation	Line
$x+2y=-4$	l
$2x-y=-4$	m
$x+2y=8$	j
$2x-y=2$	n

(a) Complete the table, by matching four of the lines to their equations.

$x + 2y = -4 \Rightarrow \quad y = -\frac{1}{2}x - 2 \quad \rightarrow l$

$2x - y = -4 \Rightarrow \quad y = 2x + 4 \quad \rightarrow m$

$x + 2y = 8 \Rightarrow \quad y = -\frac{1}{2}x + 4 \quad \rightarrow j$

$2x - y = 2 \Rightarrow \quad y = 2x - 2 \quad \rightarrow n$

(b) Hence, insert scales on the x-axis and y-axis.

Shown above

(c) Hence, find the equation of the remaining line, given that its x-intercept and y-intercept are both integers.

Equation of k: $y = -\frac{1}{2}x + 2$

or

$x + 2y = 4$