



**Geometry**

**Maths Past Exam Questions**

**Marking Schemes**

**Higher Level**

## Paper 2 – Project Maths – Section A - Q5

## Question 5

(25 marks)

- (a) In a triangle  $ABC$ , the lengths of the sides are  $a$ ,  $b$  and  $c$ .  
Using a formula for the area of a triangle, or otherwise,  
prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

$$\frac{1}{2} ac \sin \angle B = \frac{1}{2} ab \sin \angle C$$

Divide by  $\frac{1}{2} abc$

$$\frac{\sin \angle B}{b} = \frac{\sin \angle C}{c} \Rightarrow \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

OR

Case 1

$$\sin \angle B = \frac{x}{c}$$

$$\sin \angle C = \frac{x}{b}$$

$$x = c \sin \angle B$$

$$x = b \sin \angle C$$

$$b \sin \angle C = c \sin \angle B$$

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

Case 2

$$\sin(180^\circ - \angle B) = \frac{x}{c}$$

$$\sin \angle C = \frac{x}{b}$$

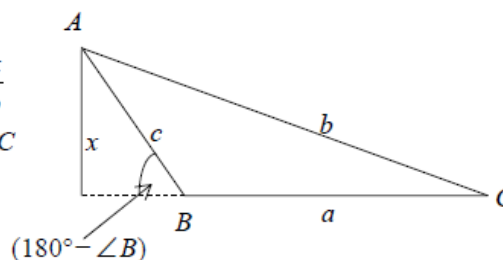
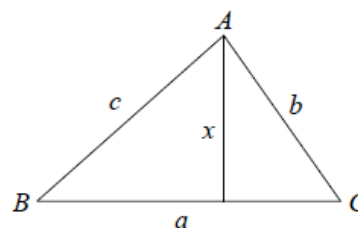
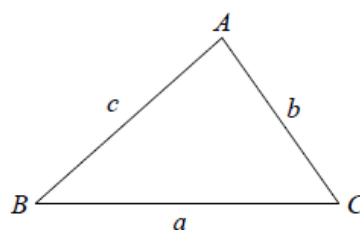
$$x = c \sin(180^\circ - \angle B)$$

$$x = b \sin \angle C$$

$$x = c \sin \angle B$$

$$b \sin \angle C = c \sin \angle B$$

$$\frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$



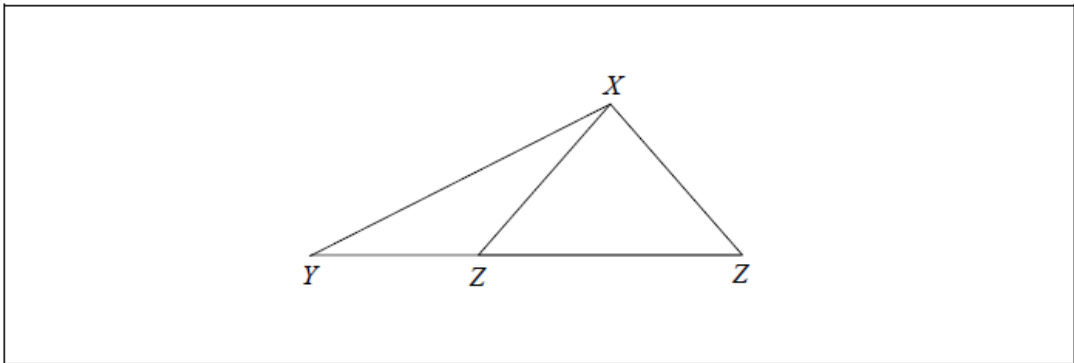
- (b) In a triangle  $XYZ$ ,  $|XY| = 5$  cm,  $|XZ| = 3$  cm and  $|\angle XYZ| = 27^\circ$ .

- (i) Find the two possible values of  $|\angle XZY|$ . Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin \angle Z} \Rightarrow \sin \angle Z = \frac{5 \sin 27^\circ}{3} = 0.756$$

$$\Rightarrow |\angle Z| = 49^\circ \text{ or } |\angle Z| = 131^\circ$$

- (ii) Draw a sketch of the triangle  $XYZ$ , showing the two possible positions of the point  $Z$ .



- (c) In the case that  $|\angle XZY| < 90^\circ$ , write down  $|\angle ZXY|$ , and hence find the area of the triangle  $XYZ$ , correct to the nearest integer.

$$|\angle ZXY| = 180^\circ - (27^\circ + 49^\circ) = 104^\circ$$

$$\Delta = \frac{1}{2}ab \sin C = \frac{1}{2}(5)(3) \sin 104^\circ = 7.27 = 7 \text{ cm}^2$$

Paper 2 – Project Maths – Section A – Q6

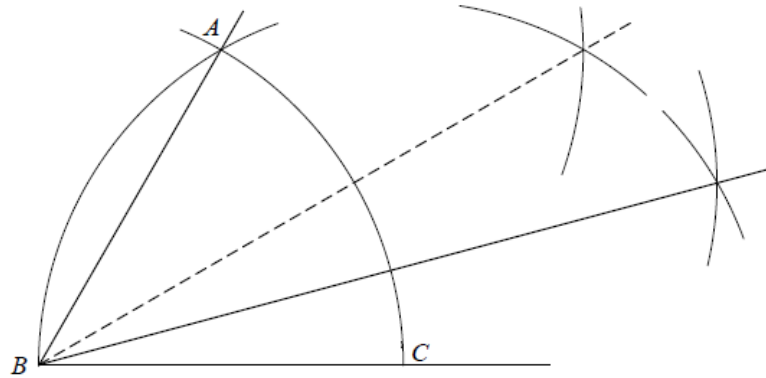
Question 6

(25 marks)

Answer either 6A or 6B.

Question 6A

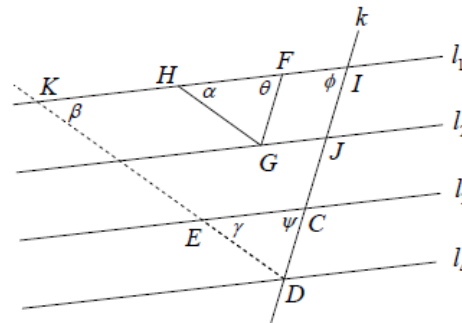
- (a) (i) Given the points  $B$  and  $C$  below, construct, without using a protractor or setsquare, a point  $A$  such that  $|\angle ABC| = 60^\circ$ .



- (ii) Hence construct, on the same diagram above, and using a compass and straight edge only, an angle of  $15^\circ$ .

Bisect  $60^\circ$  to get  $30^\circ$ ; bisect again to get  $15^\circ$  (as shown above)  
 OR  
 Construct a right angle and use it to construct  $45^\circ$  and combine with  $60^\circ$  to get  $15^\circ$ .

- (b) In the diagram,  $l_1, l_2, l_3$ , and  $l_4$  are parallel lines that make intercepts of equal length on the transversal  $k$ .  $FG$  is parallel to  $k$ , and  $HG$  is parallel to  $ED$ .



Prove that the triangles  $\triangle CDE$  and  $\triangle FGH$  are congruent.

$|CD| = |IJ|$  (given)  
 $= |FG|$  (opposite sides of parallelogram)  
 $\theta = \phi = \psi$  (corresponding angles)  
 $\alpha = \beta = \gamma$  (corresponding angles)  
 $\Rightarrow |\angle HGF| = |\angle EDC|$   
 $\therefore \triangle CDE \equiv \triangle FGH$  (ASA)

OR

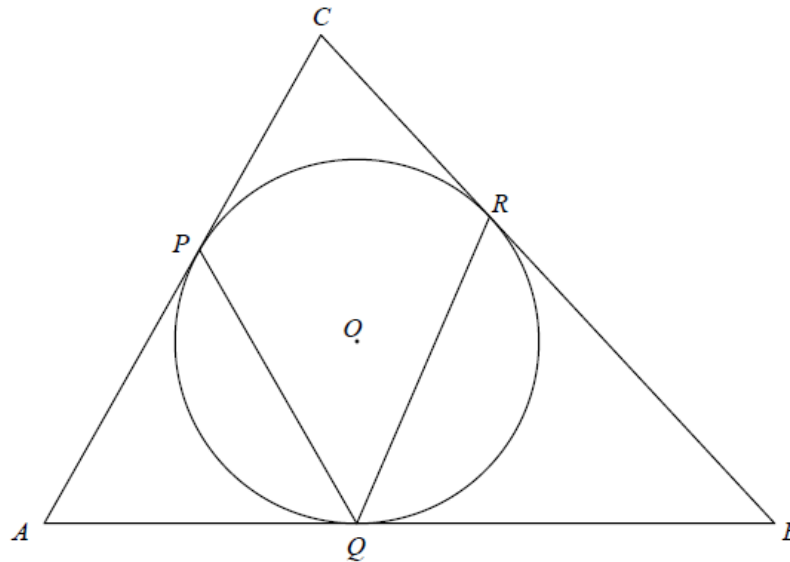
$ CD  =  LJ $ (given) $=  FG $ (opposite sides of parallelogram) $\theta = \phi = \psi$ (corresponding angles) $\alpha = \beta = \gamma$ (corresponding angles) $\therefore \triangle CDE \equiv \triangle FGH$ (ASA)	
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OR

**Question 6B**

The incircle of the triangle  $ABC$  has centre  $O$  and touches the sides at  $P, Q$  and  $R$ , as shown.

Prove that  $|\angle PQR| = \frac{1}{2}(|\angle CAB| + |\angle CBA|)$ .



$ \angle OQA  =  \angle OPA  = 90^\circ$ (radius $\perp$ tangent) $\therefore O, Q, A, P$ are concyclic. $ \angle OQP  =  \angle OAP $ (standing on same arc $OP$ ) $= \frac{1}{2} \angle PAQ $ (since $AO$ is the bisector of $\angle PAQ$ ) Similarly, $ \angle OQR  = \frac{1}{2} \angle QBR $ Adding these two gives the required result.
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OR

$$|\angle OPC| = |\angle ORC| = 90^\circ \quad (\text{radius} \perp \text{tangent})$$

$$\therefore |\angle PBR| = 180^\circ - |\angle POR| \quad (\text{angles in any quadrilateral add up to } 360^\circ)$$

$$\text{But } |\angle PBR| = 180^\circ - (|\angle CAB| + |\angle CBA|) \quad (\text{angles in a triangle})$$

$$\text{So } |\angle POR| = |\angle CAB| + |\angle CBA|$$

$$\text{But } |\angle PQR| = \frac{1}{2} |\angle POR|$$

$$\text{So } |\angle PQR| = \frac{1}{2} (|\angle CAB| + |\angle CBA|)$$

OR

$$\text{Let } OA \cap PQ = \{D\}$$

$$|OP| = |OQ| \Rightarrow |AP| = |AQ| \quad (\text{Pythagoras})$$

$$|\angle PAD| = |\angle QAD| \quad (\text{bisector})$$

$$\therefore \triangle PDA \cong \triangle QDA \quad (\text{S.A.S.})$$

$$\therefore |\angle PDA| = |\angle QDA| = 90^\circ$$

$$|\angle DAQ| = 90^\circ - |\angle DQA|$$

$$= |\angle OQD|$$

$$\therefore |\angle PAQ| = 2|\angle OQD|$$

$$\text{Similarly, } |\angle RBQ| = 2|\angle OQR|$$

Adding these two gives the required result.

OR

$$\text{Let } OA \cap PQ = \{D\}$$

$$|OP| = |OQ| \Rightarrow |AP| = |AQ| \quad (\text{Pythagoras})$$

$$|\angle APQ| = |\angle AQP| \quad (\text{isosceles triangle theorem})$$

$$\text{Similarly, } |\angle RQB| = |\angle RBQ|$$

$$|\angle AQP| + |\angle PQR| + |\angle RQB| = 180^\circ$$

$$|\angle PQR| = 180^\circ - |\angle AQP| - |\angle RQB|$$

$$|\angle CAB| = 180^\circ - 2|\angle AQP|$$

$$|\angle CBA| = 180^\circ - 2|\angle RQB|$$

$$\Rightarrow |\angle CAB| + |\angle CBA| = 360^\circ - 2[|\angle AQP| + |\angle RQB|]$$

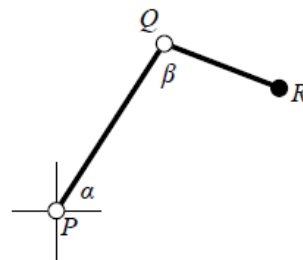
$$\Rightarrow \frac{1}{2}[|\angle CAB| + |\angle CBA|] = 180^\circ - |\angle AQP| - |\angle RQB| = |\angle PQR|$$

**Q8**

**Question 8**

**(75 marks)**

The diagram is a representation of a robotic arm that can move in a vertical plane. The point  $P$  is fixed, and so are the lengths of the two segments of the arm. The controller can vary the angles  $\alpha$  and  $\beta$  from  $0^\circ$  to  $180^\circ$ .



- (a) Given that  $|PQ| = 20$  cm and  $|QR| = 12$  cm, determine the values of the angles  $\alpha$  and  $\beta$  so as to locate  $R$ , the tip of the arm, at a point that is 24 cm to the right of  $P$ , and 7 cm higher than  $P$ . Give your answers correct to the nearest degree.

$$|PR|^2 = 7^2 + 24^2$$

$$|PR| = 25$$
  

$$25^2 = 20^2 + 12^2 - 2(20)(12)\cos\beta$$

$$\cos\beta = -0.16875$$

$$\beta \approx 100^\circ$$
  

$$12^2 = 25^2 + 20^2 - 2(25)(20)\cos(\alpha - \gamma)$$

$$\cos(\alpha - \gamma) = 0.881$$

$$\alpha - \gamma \approx 28.237^\circ$$
  

$$\tan\gamma = \frac{7}{24}$$

$$\gamma \approx 16.260^\circ$$
  

$$\therefore \alpha \approx 44^\circ$$

- (b) In setting the arm to the position described in part (a), which will cause the greater error in the location of  $R$ : an error of  $1^\circ$  in the value of  $\alpha$  or an error of  $1^\circ$  in the value of  $\beta$ ?

Justify your answer. You may assume that if a point moves along a circle through a small angle, then its distance from its starting point is equal to the length of the arc travelled.

Ans:  $\alpha$

Reason:  $1^\circ$  error in  $\alpha$  causes  $R$  to move along an arc of radius 25.  
 $1^\circ$  error in  $\beta$  causes  $R$  to move along an arc of radius 12.

So, since  $l = r\theta$ , and  $\theta$  is the same in each case, the point moves further in the first case.

- (c) The answer to part (b) above depends on the particular position of the arm. That is, in certain positions, the location of  $R$  is more sensitive to small errors in  $\alpha$  than to small errors in  $\beta$ , while in other positions, the reverse is true. Describe, with justification, the conditions under which each of these two situations arises.

More sensitive to errors in  $\alpha$  when  $|PR| > 12$

More sensitive to errors in  $\beta$  when  $|PR| < 12$

The condition  $|PR| > 12$

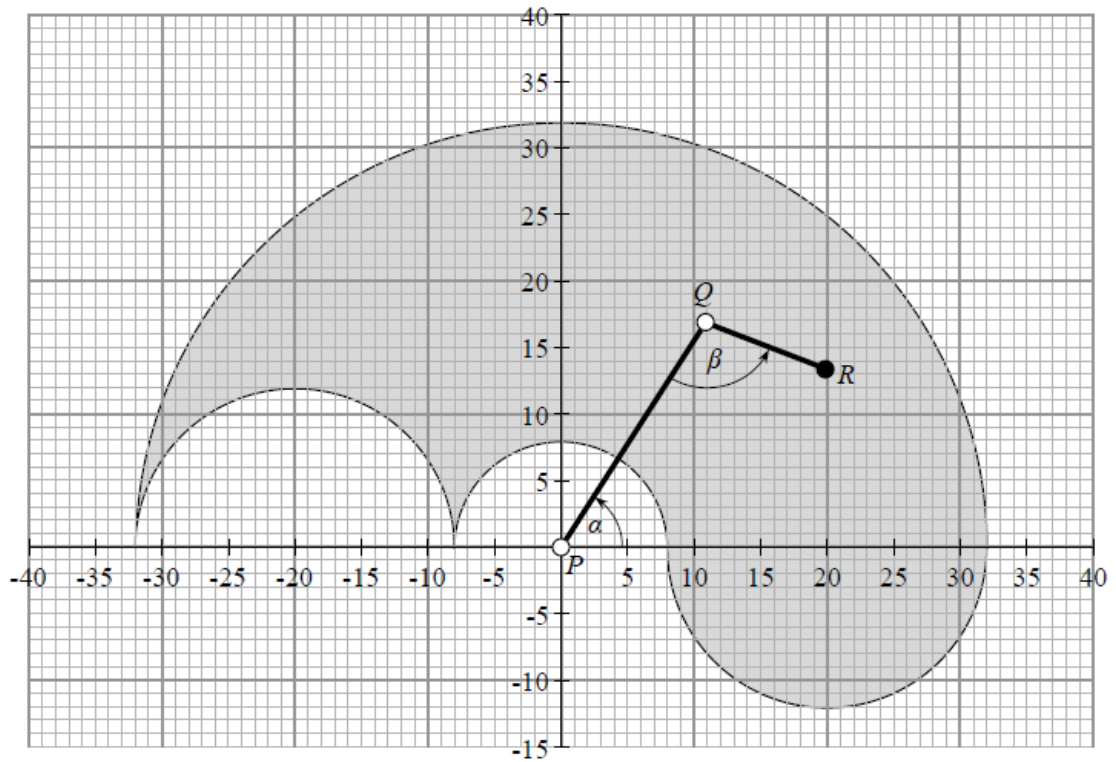
is true whenever

$$\beta > \cos^{-1}\left(\frac{5}{6}\right) \approx 33.6^\circ$$

(Borderline case is when  $\Delta PQR$  is isosceles with  $|QR| = |RP|$ .)



- (d) Illustrate the set of all possible locations of the point  $R$  on the coordinate diagram below. Take  $P$  as the origin and take each unit in the diagram to represent a centimetre in reality. Note that  $\alpha$  and  $\beta$  can vary only from  $0^\circ$  to  $180^\circ$ .

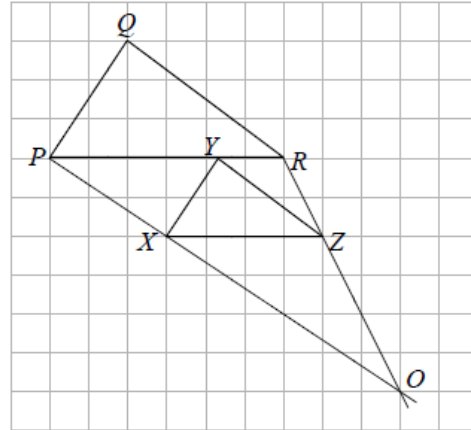


Paper 2 – Project Maths – Section A – Q4

Question 4

(25 marks)

Two triangles are drawn on a square grid as shown. The points  $P, Q, R, X,$  and  $Z$  are on vertices of the grid, and the point  $Y$  lies on  $[PR]$ . The triangle  $PQR$  is an enlargement of the triangle  $XYZ$ .



- (a) Calculate the scale factor of the enlargement, showing your work.

$$\frac{|PR|}{|XZ|} = \frac{6}{4} = \frac{3}{2}$$

- (b) By construction or otherwise, locate the centre of enlargement on the diagram above.

Shown as  $O$  above.

- (c) Calculate  $|YR|$  in grid units.

$ Y'Z  = \frac{2}{3} Q'R  = \frac{2}{3}(4) = \frac{8}{3}$ $ YR  = \frac{8}{3} - 1 = \frac{5}{3}$	$\tan \alpha = \frac{3}{2}$ $\alpha = \beta$ $\frac{2}{ XY' } = \frac{3}{2}$ $ XY'  = \frac{4}{3}$ $ YR  = 3 - \frac{4}{3} = \frac{5}{3}$
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