Geometry (Proofs)

Maths Past Exam Questions

Marking Schemes

Higher Level

2013

Paper Two- Project Maths - Section A - Question 6

Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Complete each of the following statements.

(i) The circumcentre of a triangle is the point of intersection of

the perpendicular bisectors of the sides of the triangle

(ii) The incentre of a triangle is the point of intersection of

the bisectors of the angles of the triangle

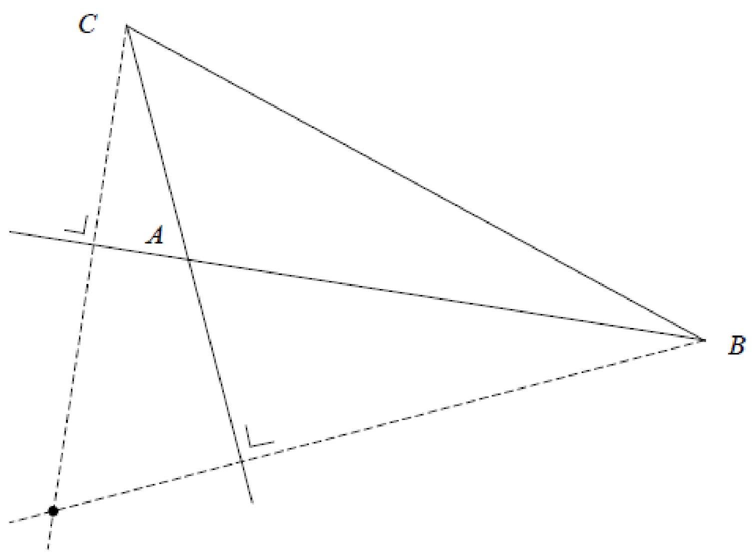
(iii) The centroid is the point of intersection of

the medians of the triangle

(b) In an equilateral triangle, the circumcentre, the incentre and the centroid are all in the same place. Explain why this is the case.

In an equilateral triangle the medians are perpendicular to the opposite sides and bisect the angles. Therefore, the perpendicular bisectors of the sides, the bisectors of the angles and the median are the same line and intersect at one point.

(c) Construct the orthocentre of the triangle *ABC* below. Show all construction lines clearly.



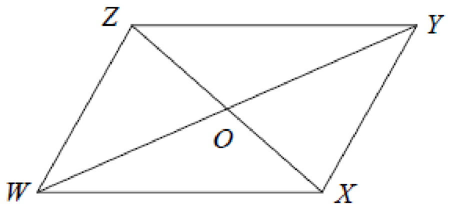
OR

Question 6B

(a) A quadrilateral (four sided figure) has two sides which are parallel and equal in length. Prove that the quadrilateral is a parallelogram.

In the quadrilateral $WXYZ$, $WX \parallel ZY$ and $|WX| = |ZY|$

To Prove: $WXYZ$ is a parallelogram.



Join Z to X and Y to W

Proof:

In ΔZOY and ΔOWX ,

$|ZY| = |WX|$

$|\angle ZYO| = |\angle OWX|$ ... $ZY \parallel WX$

$|\angle YZO| = |\angle OXW|$ ... $ZY \parallel WX$

Hence, ΔZOY and ΔOWX are congruent since AAS

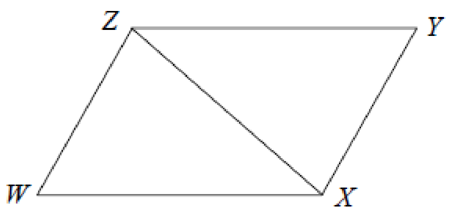
Hence, $|ZO| = |OX|$ and $|YO| = |OW|$

Hence, the diagonals of $WXYZ$ bisect each other $\Rightarrow$ $WXYZ$ is a parallelogram.

OR

In the quadrilateral $WXYZ$, $WX \parallel ZY$ and $|WX| = |ZY|$

To Prove: $WXYZ$ is a parallelogram.



Join Z to X

Proof:

In ΔWXZ and ΔYZX ,

$|WX| = |ZY|$

$|\angle YZX| = |\angle WXZ|$ ... $ZY \parallel WX$

$|ZX| = |ZX|$ ... common to both

Hence, ΔWXZ and ΔZXY are congruent since SAS

$\Rightarrow WZ$ and XY parallel

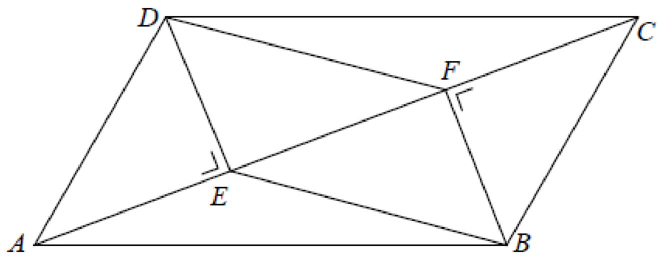
$\Rightarrow$ $WXYZ$ is a parallelogram.

(b) In the parallelogram $ABCD$,

DE is perpendicular to AC.

BF is perpendicular to AC.

Prove that $EBFD$ is a parallelogram.



In the parallelogram $ABCD$,

$DE \perp AC$ and $AC \perp BF \Rightarrow DE \parallel BF$.

In the parallelogram $ABCD$,

area of ΔDAC = area of $\Delta ABC \Rightarrow |DE| = |BF|$.

$DE \parallel BF$ and $|DE| = |BF| \Rightarrow EBFD$ is a parallelogram.

2011

Paper 2 – Project Maths – Section A – Q6

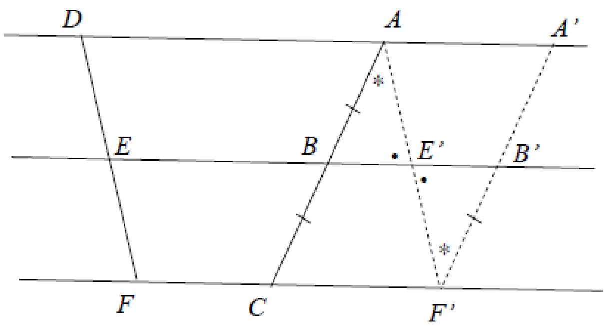
Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:



Given: $AD \parallel BE \parallel CF$, as in the diagram, with $|AB| = |BC|$

To prove: $|DE| = |EF|$

Construction:

Draw $AE' \parallel DE$, cutting EB at E' and CF at F'
Draw $F'B' \parallel AB$, cutting EB at B', as in the diagram.

Proof:

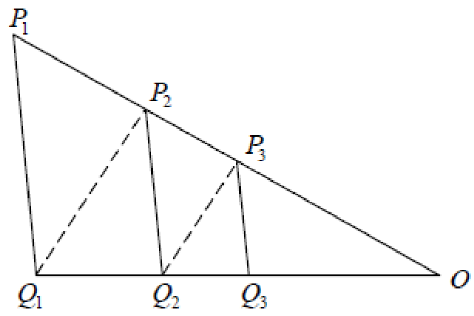
$\lvert B'F' \rvert = \lvert BC \rvert$	(opposite sides in a parallelogram)
$= \lvert AB \rvert$	(by assumption)
$\lvert \angle BAE' \rvert = \lvert \angle E'F'B' \rvert$	(alternate angles)
$\lvert \angle AE'B \rvert = \lvert \angle F'E'B' \rvert$	(vertically opposite angles)
$\therefore\ \Delta ABE'$ is congruent to $\Delta F'B'E'$	(ASA)
Therefore $\lvert AE' \rvert = \lvert F'E' \rvert$.	
But $\lvert AE' \rvert = \lvert DE \rvert$ and $\lvert F'E' \rvert = \lvert FE \rvert$	(opposite sides in a parallelogram)
$\therefore\ \lvert DE \rvert = \lvert EF \rvert$.	

OR

Question 6B

In the diagram, P_1Q_1, P_2Q_2, and P_3Q_3 are parallel and so also are Q_1P_2 and Q_2P_3.

Prove that $|P_1Q_1| \times |P_3Q_3| = |P_2Q_2|^2$.



$$\frac{|OP_3|}{|OP_2|} = \frac{|OQ_2|}{|OQ_1|} \qquad (P_3Q_2 \parallel P_2Q_1)\ ^*$$

$$\frac{|OP_2|}{|OP_1|} = \frac{|OQ_2|}{|OQ_1|} \qquad (P_2Q_2 \parallel P_1Q_1)$$

$$\therefore \frac{|OP_3|}{|OP_2|} = \frac{|OP_2|}{|OP_1|}$$

$$\frac{|OP_2|}{|OP_1|} = \frac{|P_2Q_2|}{|P_1Q_1|} \qquad (\textit{Similar triangles})$$

$$\frac{|OP_3|}{|OP_2|} = \frac{|P_3Q_3|}{|P_2Q_2|} \qquad (\textit{Similar triangles})$$

$$\therefore \frac{|P_2Q_2|}{|P_1Q_1|} = \frac{|P_3Q_3|}{|P_2Q_2|}$$

$$\therefore |P_1Q_1||P_3Q_3| = |P_2Q_2|^2$$