Graphs of Functions

Maths Past Exam Questions

Marking Schemes

Higher Level

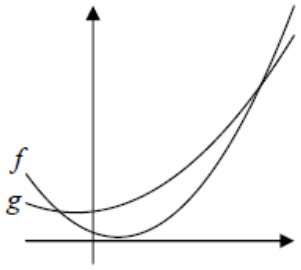
2012

Paper 1 – Project Maths – Q5

Question 5 **(25 marks)**

The functions f and g are defined for $x \in \mathbb{R}$ as

$$f : x \mapsto 2x^2 - 3x + 2 \quad \text{and}$$
$$g : x \mapsto x^2 + x + 7 .$$



(a) Find the co-ordinates of the two points where the curves $y = f(x)$ and $y = g(x)$ intersect.

$$f(x) = g(x)$$
$$2x^2 - 3x + 2 = x^2 + x + 7$$
$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$
$$x = -1, \quad x = 5.$$

$$f(-1) = 7 \Rightarrow (-1, 7)$$
$$f(5) = 37 \Rightarrow (5, 37)$$

(b) Find the area of the region enclosed between the two curves.

$$A = \int_{-1}^{5} (g(x) - f(x))dx$$
$$= \int_{-1}^{5} \left(-x^2 + 4x + 5\right) dx$$
$$= \left[\frac{-x^3}{3} + 2x^2 + 5x\right]_{-1}^{5}$$
$$= \left(\tfrac{-125}{3} + 50 + 25\right) - \left(\tfrac{1}{3} + 2 - 5\right)$$
$$= 36.$$