Probability

Maths Past Exam Questions

Marking Schemes

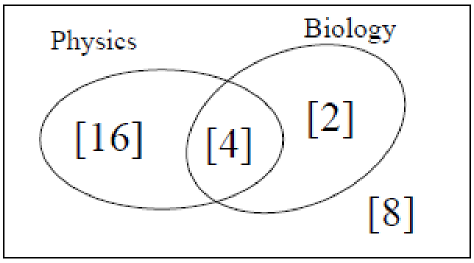
Higher Level

2013

Paper 2 – Project Maths – Section A – Q1 B

(b) In a class of 30 students, 20 study Physics, 6 study Biology and 4 study both Physics and Biology.

(i) Represent the information on the Venn Diagram.



A student is selected at random from this class.
The events E and F are:

E: The student studies Physics

F: The student studies Biology.

(ii) By calculating probabilities, investigate if the events E and F are independent.

$P(E \cap F) = \frac{4}{30}$

$P(E) \times P(F) = \frac{20}{30} \times \frac{6}{30} = \frac{4}{30}$

$P(E \cap F) = P(E) \times P(F) \Rightarrow$ E and F are independent events

2012

Paper 2 – Project Maths – Section A – Q4

Question 4 **(25 marks)**

A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.

(a) What assumption(s) must be made in order to regard this as a sequence of Bernoulli trials?

Trials are independent of each other.
Probability of success is the same each time.

[Only two outcomes …. (Given)]
[Finite number of throws…… (Given)]

(b) Based on such assumption(s), find, correct to three decimal places, the probability that:

(i) she scores on exactly four of the six shots

$P(X = 4) = {}^6C_4(0{\cdot}6)^4(0{\cdot}4)^2 = 0{\cdot}31104$

$= 0{\cdot}311$ to three decimal places.

(ii) she scores for the second time on the fifth shot.

Exactly one success among first four throws, followed by success on fifth:

$\left({}^4C_1(0{\cdot}6)(0{\cdot}4)^3\right)(0{\cdot}6) = 0{\cdot}09216$

$= 0{\cdot}092$ to three decimal places.

2011

Paper 2 – Project Maths – Section A – Q1 B

(b) There are 16 girls and 8 boys in a class. Half of these 24 students study French. The probability that a randomly selected girl studies French is 1·5 times the probability that a randomly selected boy studies French. How many of the boys in the class study French?

Let x = number of boys, who study French.

$\therefore 12 - x$ = number of girls who study French.

$$\frac{12-x}{16} = 1{\cdot}5\left(\frac{x}{8}\right)$$

$$96 - 8x = 24x$$

$$32x = 96$$

$$x = 3$$

Three boys study French.

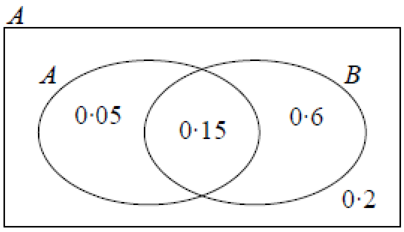
2010

Paper 2 – Project Maths – Section A – Q1

Question 1 **(25 marks)**

Two events A and B are such that $P(A) = 0{\cdot}2$, $P(A \cap B) = 0{\cdot}15$ and $P(A' \cap B) = 0{\cdot}6$.

(a) Complete this Venn diagram.



(b) Find the probability that neither A nor B happens.

$$0{\cdot}2.$$

or

$$P(A \cup B)' = 1 - P(A \cup B) = 1 - (0{\cdot}05 + 0{\cdot}15 + 0{\cdot}6) = 0{\cdot}2$$

(c) Find the conditional probability $P(A \mid B)$.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{0{\cdot}15}{0{\cdot}75} = 0{\cdot}2.$$

(d) State whether A and B are independent events and justify your answer.

A and B are independent events as, $P(A \mid B) = P(A) = 0{\cdot}2$.

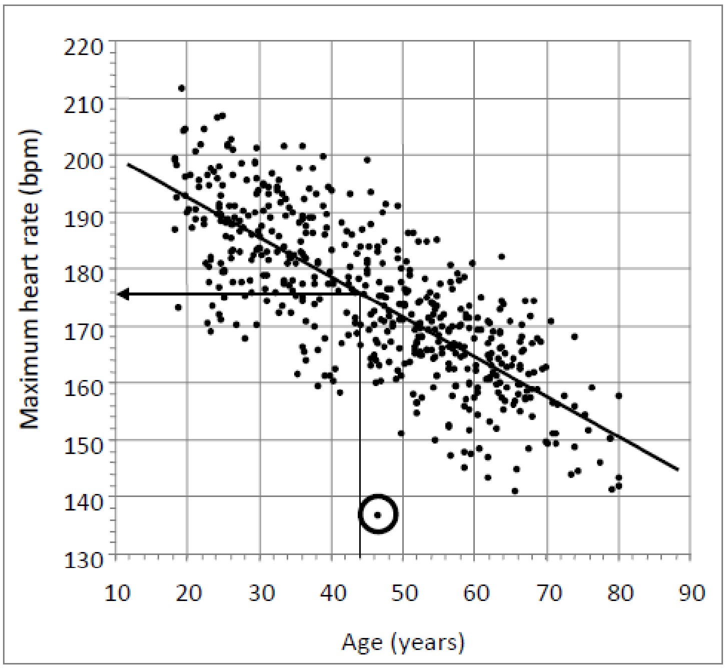
or

A and B are independent events as, $P(A)P(B) = (0{\cdot}2)(0{\cdot}75) = 0{\cdot}15 = P(A \cap B)$.

Paper 2 – Project Maths – Section B – Q7

Question 7 **Probability and Statistics** **(50 marks)**

A person's *maximum heart rate* is the highest rate at which their heart beats during certain extreme kinds of exercise. It is measured in beats per minute (bpm). It can be measured under controlled conditions. As part of a study in 2001, researchers measured the maximum heart rate of 514 adults and compared it to each person's age. The results were like those shown in the scatter plot below.



Source: Simulated data based on: Tanaka H, Monaghan KD, and Seals DR. *Age-predicted maximal heart rate revisited*, J. Am. Coll. Cardiol. 2001;37;153-156.

(a) From the diagram, estimate the correlation coefficient. Answer: – 0·75

(b) Circle the *outlier* on the diagram and write down the person's age and maximum heart rate.

Age = 47 years Max. heart rate = 137 bpm

(c) The line of best fit is shown on the diagram. Use the line of best fit to estimate the maximum heart rate of a 44-year-old person. Answer: 176 bpm

(d) By taking suitable readings from the diagram, calculate the slope of the line of best fit.

Possible Readings

$(10, 200)$ and $(90, 144)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{144 - 200}{90 - 10} = -\frac{-56}{80} = -\frac{7}{10} \text{ or } m = -0{\cdot}7.$$

(e) Find the equation of the line of best fit and write it in the form: $MHR = a - b \times (\text{age})$, where *MHR* is the maximum heart rate.

$$y - y_1 = m(x - x_1)$$
$$y - 200 = -0{\cdot}7(x - 10)$$
$$y = -0{\cdot}7x + 207$$
$$MHR = 207 - 0{\cdot}7 \times (age)$$

(f) The researchers compared their new rule for estimating maximum heart rate to an older rule. The older rule is: $MHR = 220 - \text{age}$. The two rules can give different estimates of a person's maximum heart rate. Describe how the level of agreement between the two rules varies according to the age of the person. Illustrate your answer with two examples.

For young adults the old rule gives a greater *MHR* than the new rule.

Adult aged 20

MHR = 220 – 20 = 200 bpm (Old rule)

$MHR = 207 - 0{\cdot}7(20) = 193$ bpm (New Rule)

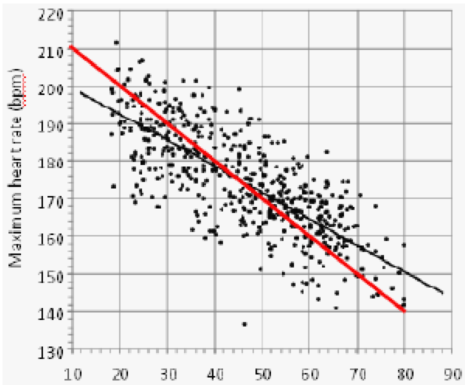
Towards middle age there is a greater agreement between the rules.

For older people the new rule gives a greater *MHR* than the old rule.

Adult aged 70

MHR = 220 – 70 = 150 bpm

$MHR = 207 - 0{\cdot}7(70) = 158$ bpm



(g) A particular exercise programme is based on the idea that a person will get most benefit by exercising at 75% of their estimated *MHR*. A 65-year-old man has been following this programme, using the old rule for estimating *MHR*. If he learns about the researchers' new rule for estimating *MHR*, how should he change what he is doing?

He should exercise a bit more intensely.

Using the old rule he exercises to 75% of (220 – 65) = 116 bpm.

Using the new rule he can exercise to 75% of $(207 - 0{\cdot}7 \times 65)$ = 121 bpm.

Paper 2 – Project Maths – Section B – Q9

Question 9A **Probability and Statistics** **(50 marks)**

A factory manufactures aluminium rods. One of its machines can be set to produce rods of a specified length. The lengths of these rods are normally distributed with mean equal to the specified length and standard deviation equal to 0·2 mm.

The machine has been set to produce rods of length 40 mm.

(a) What is the probability that a randomly selected rod will be less than 39·7 mm in length?

$$P(X < 39{\cdot}7) = P\left(Z < \frac{39{\cdot}7 - 40}{0{\cdot}2}\right) = P(Z < -1{\cdot}5)$$
$$= P(z > 1{\cdot}5)$$
$$= 1 - P(Z \leq 1{\cdot}5)$$
$$= 1 - 0{\cdot}9332$$
$$= 0{\cdot}0668$$

(b) Five rods are selected at random. What is the probability that at least two of them are less than 39·7 mm in length?

Binomial distribution with $n = 5$, $p = 0{\cdot}0668$, $q = 0{\cdot}9332$.

$$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X = 1) + P(X = 0)]$$
$$= 1 - \left[\binom{5}{1}(0{\cdot}0668)(0{\cdot}9332)^4 + \binom{5}{0}(0{\cdot}9332)^5\right]$$
$$= 0{\cdot}03895.$$

Or

$$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$
$$= \binom{5}{2}(0{\cdot}0668)^2(0{\cdot}9332)^3 + \binom{5}{3}(0{\cdot}0668)^3(0{\cdot}9332)^2 + \binom{5}{4}(0{\cdot}0668)^4(.9332) + \binom{5}{5}(0{\cdot}0668)^5$$
$$= 0{\cdot}03895$$

(c) The operators want to check whether the setting on the machine is still accurate. They take a random sample of ten rods and measure their lengths. The lengths in millimetres are:

39·5	40·0	39·7	40·2	39·8
39·7	40·2	39·9	40·1	39·6

Conduct a hypothesis test at the 5% level of significance to decide whether the machine's setting has become inaccurate. You should start by clearly stating the null hypothesis and the alternative hypothesis, and finish by clearly stating what you conclude about the machine.

$H_0 : \mu = 40$ mm (null hypothesis)
$H_1 : \mu \neq 40$ mm (alternative hypothesis)

$$\sigma_{\bar{x}} = \frac{0{\cdot}2}{\sqrt{10}} = 0{\cdot}0632456$$

Observed value of $\bar{x} = 39{\cdot}87$

$$\therefore \text{ Observed } z = \frac{39{\cdot}87 - 40}{0{\cdot}0632456} = -2{\cdot}055$$

The critical values for the test are $\pm 1{\cdot}96$

As $-2{\cdot}055 < -1{\cdot}96$, we reject the null hypothesis at the 5% level of significance and we conclude that the machine setting has become inaccurate.