Trigonometry

Maths Past Exam Questions

Marking Schemes

Higher Level

2013

Paper 2 – Project Maths – Section A Q5

Question 5 **(25 marks)**

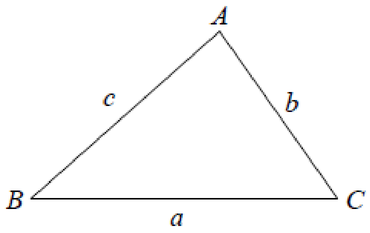
(a) In a triangle ABC, the lengths of the sides are a, b and c.
Using a formula for the area of a triangle, or otherwise, prove that

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}.$$

$$\frac{1}{2}ac\sin\angle B = \frac{1}{2}ab\sin\angle C$$

Divide by $\frac{1}{2}abc$

$$\frac{\sin\angle B}{b} = \frac{\sin\angle C}{c} \Rightarrow \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



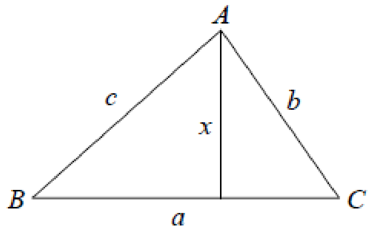
OR
Case 1

$\sin\angle B = \frac{x}{c}$ $\qquad$ $\sin\angle C = \frac{x}{b}$

$x = c\sin\angle B$ $\qquad$ $x = b\sin\angle C$

$b\sin\angle C = c\sin\angle B$

$$\frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



Case 2

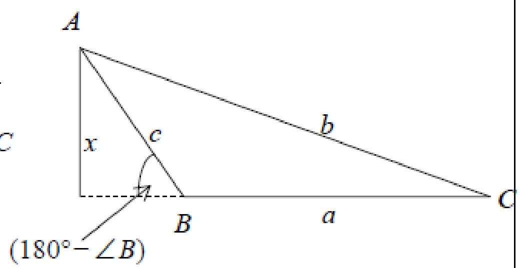
$\sin(180^\circ - \angle B) = \frac{x}{c}$ $\qquad$ $\sin\angle C = \frac{x}{b}$

$x = c\sin(180^\circ - \angle B)$ $\qquad$ $x = b\sin\angle C$

$x = c\sin\angle B$

$b\sin\angle C = c\sin\angle B$

$$\frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$



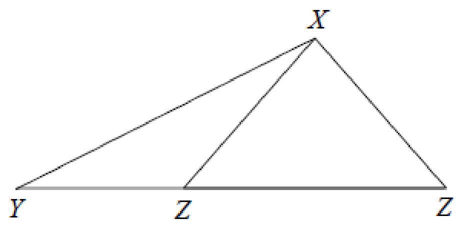
(b) In a triangle XYZ, $|XY| = 5$ cm, $|XZ| = 3$ cm and $|\angle XYZ| = 27^\circ$.

(i) Find the two possible values of $|\angle XZY|$. Give your answers correct to the nearest degree.

$$\frac{3}{\sin 27^\circ} = \frac{5}{\sin\angle Z} \Rightarrow \sin\angle Z = \frac{5\sin 27^\circ}{3} = 0\cdot 756$$

$$\Rightarrow |\angle Z| = 49^\circ \quad \text{or} \quad |\angle Z| = 131^\circ$$

(ii) Draw a sketch of the triangle *XYZ*, showing the two possible positions of the point *Z*.



(c) In the case that $|\angle XZY| < 90°$, write down $|\angle ZXY|$, and hence find the area of the triangle *XYZ*, correct to the nearest integer.

$$|\angle ZXY| = 180° - (27° + 49°) = 104°$$

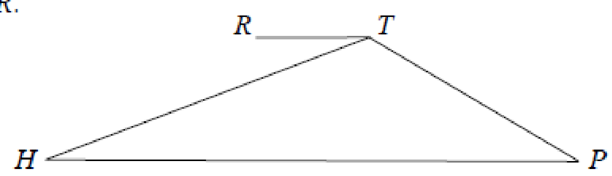
$$\Delta = \tfrac{1}{2}ab\sin C = \tfrac{1}{2}(5)(3)\sin 104° = 7 \cdot 27 = 7 \text{ cm}^2$$

Paper 2 – Project Maths – Section B Q8

(b) The point T is directly East of the point R.

$|HT| = 110$ km and $|TP| = 80$ km.

Find $|RT|$.

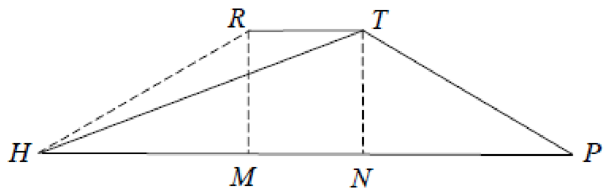


Some possible solutions:

$\cos 36° = \frac{|HM|}{80}$

$\Rightarrow |HM| = 80\cos 36°$

$= 64 \cdot 72$ km



$|NP| = 64 \cdot 72$

$|RT| = |MN| = 168 - 2(64 \cdot 72) = 38 \cdot 56$ km

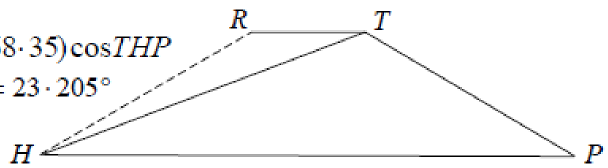
OR

Taking ΔHTP

$80^2 = 110^2 + 168 \cdot 35^2 - 2(110)(168 \cdot 35)\cos THP$

$\Rightarrow \cos \angle THP = \cdot 9191 \Rightarrow \angle THP = 23 \cdot 205°$

$\Rightarrow \angle RHT = 12 \cdot 795°$



Taking ΔHRT

$|RT|^2 = 110^2 + 80^2 - 2(110)(80)\cos 12 \cdot 795°$

$|RT|^2 = 1337 \cdot 0 \Rightarrow |RT| = 36 \cdot 56$ km

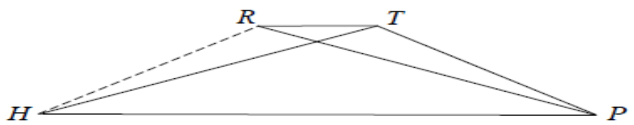
OR

$|\angle RPH| = |\angle THP| = 20^0$

$\Rightarrow |\angle RHT| = 16^0$

$|RT|^2 = 110^2 + 80^2 - 2(110)(80)\cos 16°$

$|RT|^2 = 1581 \cdot 824 \Rightarrow |RT| = 39 \cdot 77$ km



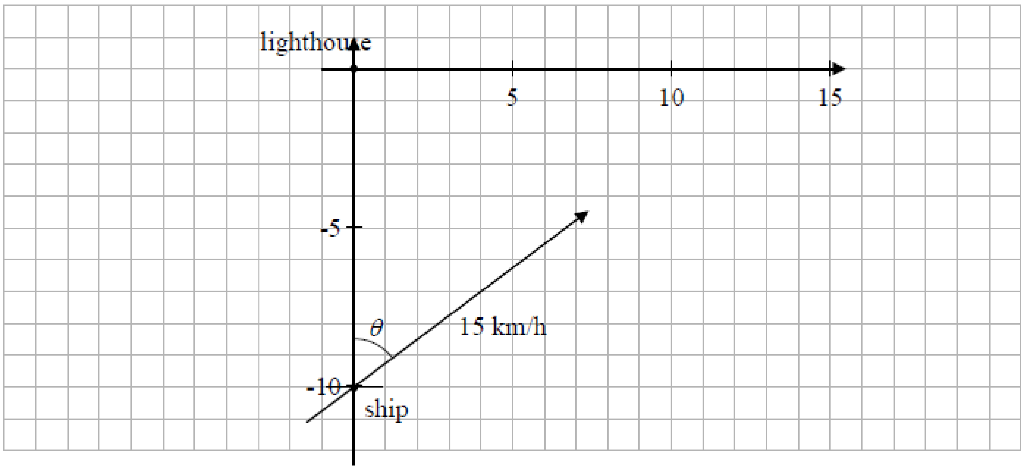
2010

Paper 2 – Project Maths – Section B Q8

Question 8 **Geometry and Trigonometry** **(50 marks)**

A ship is 10 km due South of a lighthouse at noon.

The ship is travelling at 15 km/h on a bearing of θ, as shown below, where $\theta = \tan^{-1}\left(\frac{4}{3}\right)$.



(a) On the diagram above, draw a set of co-ordinate axes that takes the lighthouse as the origin, the line East-West through the lighthouse as the x-axis, and kilometres as units.

(b) Find the equation of the line along which the ship is moving.

$$\tan\theta = \frac{4}{3}$$

$$\therefore m = \frac{3}{4}$$

$$y + 10 = \frac{3}{4}(x - 0)$$

$$4y + 40 = 3x$$

$$3x - 4y - 40 = 0$$

Or

$$y = mx + c$$

$$y = \frac{3}{4}x - 10$$

(c) Find the shortest distance between the ship and the lighthouse during the journey.

$$\sin\theta = \frac{d}{10}$$

$$\frac{4}{5} = \frac{d}{10}$$

$$5d = 40$$

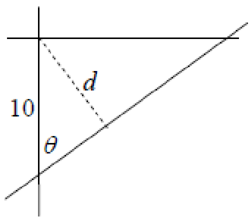
$$x = 8 \text{ km}$$

Or

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$d = \frac{|3(0) - 4(0) - 40|}{\sqrt{3^2 + (-4)^2}}$$

$$d = 8 \text{ km.}$$



(d) At what time is the ship closest to the lighthouse?

$$\tan\theta = \frac{8}{x}$$

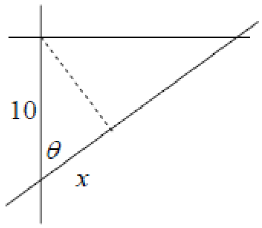
$$\frac{4}{3} = \frac{8}{x}$$

$$4x = 24$$

$$x = 6 \text{ km.}$$

$$\text{Time} = \frac{6}{15} = 0{\cdot}4 \text{ hours} = 24 \text{ minutes.}$$

$\therefore$ closest to the lighthouse at 12:24 pm



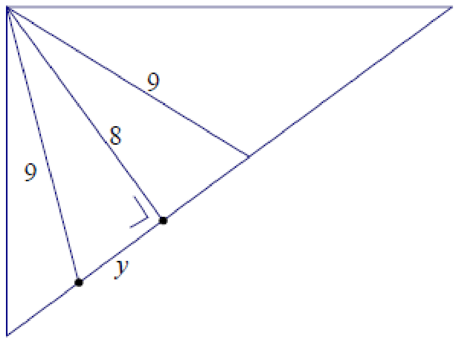
(e) Visibility is limited to 9 km. For how many minutes in total is the ship visible from the lighthouse?

$$8^2 + y^2 = 9^2$$

$$y^2 = 81 - 64$$

$$y^2 = 17$$

$$y = \sqrt{17}$$



Distance travelled by the ship while visible from the lighthouse is $2y = 2\sqrt{17}$ km.

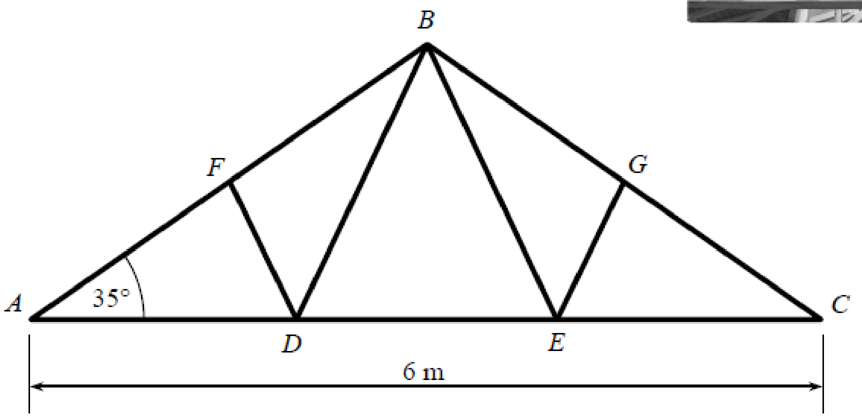
$$\text{Time} = \frac{2\sqrt{17}}{15} \text{ hours.}$$

$$= 8\sqrt{17} \text{ minutes or } 32{\cdot}98 \text{ minutes} \approx 33 \text{ minutes.}$$

Paper 2 – Project Maths – Section B Q9 B

(b) Roofs of buildings are often supported by frameworks of timber called *roof trusses*.

A quantity surveyor needs to find the total length of timber needed in order to make the triangular truss shown below.



The length of $[AC]$ is 6 metres, and the pitch of the roof is 35°, as shown.
$|AD| = |DE| = |EC|$ and $|AF| = |FB| = |BG| = |GC|$.

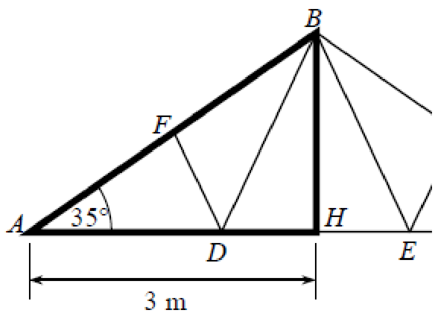
(i) Calculate the length of $[AB]$, in metres, correct to two decimal places.

$|AH| = 3$ m

$$\cos 35° = \frac{3}{|AB|}$$

$$|AB| = \frac{3}{\cos 35°} \approx 3{\cdot}66232$$

$|AB| = 3{\cdot}66$ m (to 2 decimal places)



(ii) Calculate the total length of timber required to make the truss.

$|FD|^2 = 1{\cdot}83^2 + 2^2 - 2(1{\cdot}83)(2)\cos 35°$
$= 1{\cdot}352707.$
$|FD| = 1{\cdot}163$ m

$|BD|^2 = 2^2 + 3{\cdot}66^2 - 2(2)(3{\cdot}66)\cos 35°$
$= 5{\cdot}403214.$
$|BD| = 2{\cdot}325$ m.

OR Similar triangles $\Rightarrow |BE| = 2|FD|$

Total length required $= 6 + 2(3{\cdot}662) + 2(1{\cdot}163) + 2(2{\cdot}325) = 20{\cdot}296 = 20{\cdot}3$ m.